The Radian and Angles

KNOW	DO	UNDERSTAND
How to recognize angles in radians. What quadrant an angle is in.	Determine coterminal angles to a given angle. Determine the trig ratios of an angle. How to use the unit circle and special triangles	*Function Characteristics*: Why the (x, y) coordinate on the unit circle is $(\cos\theta, \sin\theta)$

Vocab & Notation

- Radian
- Co-terminal
- Special Triangle
- Unit Circle
- Secant, Cosecant, Cotanget

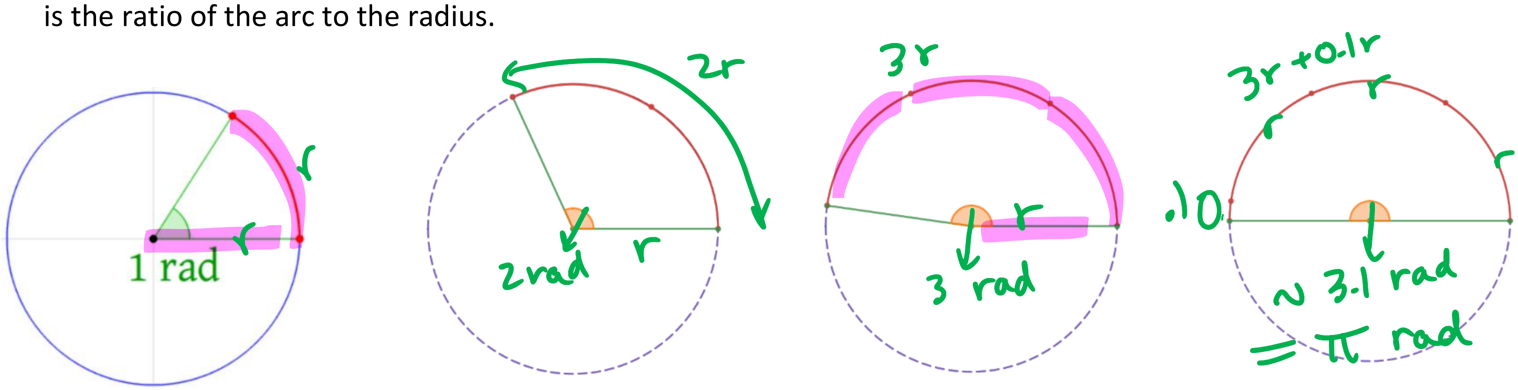
Why is there 360° in a full rotation? 400 grad

There is ~ 360 days in a year

* 360 has a lot of factors

You think trigonometry is about triangles, but really it is about circles.

Definition: One **radian** is equal to the angle made when the arc of a circle is equal to the radius. In general, it is the ratio of the arc to the radius.



$$\text{Angle (radians)} = \frac{\text{Arc Length}}{\text{radius}} \quad \Rightarrow \quad \theta = \frac{a}{r}$$

Where θ is the angle in radians. This may look like a formula, but it is the definition of the radian. This is similar to how π is defined as the ratio between the circumference and the diameter of a circle.

If we go all around the circle, then: arc = circumferance = $2\pi r$

radius = r

angle = $\frac{2\pi r}{r} = 2\pi = 360°$

* $\pi = \frac{\text{circum}}{\text{diam.}}$

Example:

$\frac{\pi}{6} = \frac{1}{6}\pi = \frac{1}{2}\frac{\pi}{3} = 30^\circ$

$\frac{\pi}{4} \times \frac{360^\circ}{2\pi} = \frac{360^\circ}{8} = 45^\circ$

$\frac{\pi}{2} = \frac{1}{2}\pi = \frac{1}{2}180^\circ = 90^\circ$

$\frac{\pi}{3} = \frac{1}{3}\pi = 60^\circ$

$100^\circ \times \frac{\pi}{180^\circ} = \frac{5}{9}\pi$

$250^\circ \times \frac{\pi}{180} = \frac{25}{18}\pi \sim 1.4\pi = 4.36$

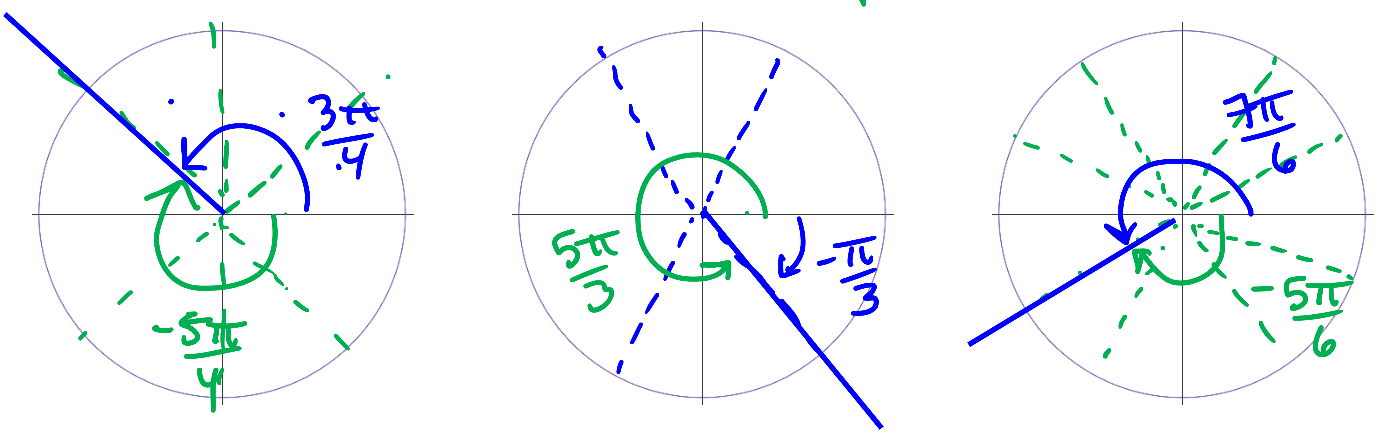
**Recall that positive angles move counter-clockwise around the circle, and negative angles move clockwise.

Definition: Angles are considered **co-terminal** if they have the same terminal arms.

Any multiple of 360° or 2π will wrap around back to the same terminal arm so we say the **general form** (which represents all the possible solutions) as

$$\boxed{\theta + 2\pi n, \quad n \in \mathbb{Z}}$$

Examples:



Angle	General Form	Domain	Coterminal Angles
$\frac{19\pi}{4}$	$-\frac{5\pi}{4} + \frac{8\pi}{4}n, n \in \mathbb{Z}$	$\underbrace{-2\pi \leq \theta < 4\pi}_{3 \text{ rotation}}$	$-\frac{5\pi}{4}$ (n=0), $\frac{3\pi}{4}$ (n=1), $\frac{11\pi}{4}$ (n=2)
$\frac{11\pi}{3}$	$\frac{5\pi}{3} + \frac{6\pi n}{3}, n \in \mathbb{Z}$	$-3\pi \leq \theta < \pi$	$-\pi/3$, $-7\pi/3$
$\frac{-17\pi}{6}$	$\frac{7\pi}{6} + \frac{12\pi}{6}n, n \in \mathbb{Z}$	$0 \leq \theta < 8\pi$	$\frac{7\pi}{6}$, $\frac{19\pi}{6}$, $\frac{31\pi}{6}$, $\frac{43\pi}{6}$

soh cah toa

When defining the angle around a circle, it is useful to think of that circle on a grid centered at the origin. Such a circle is defined by the equation:

$$x^2 + y^2 = r^2$$

Circle

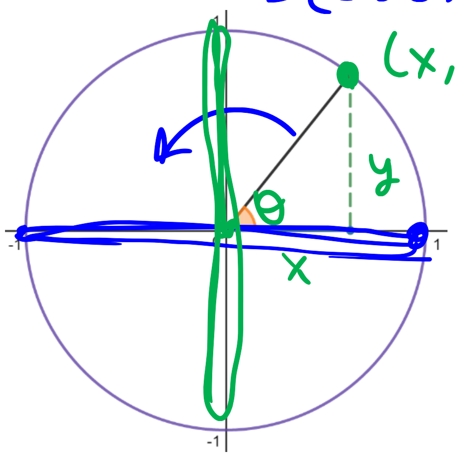
Where r is the radius of the circle.

As the angle does not change as the radius changes the **unit circle** is the circle with radius 1, centered about the origin.

$$\boxed{x^2 + y^2 = 1}$$

$= (\cos\theta, \sin\theta)$

(x, y)

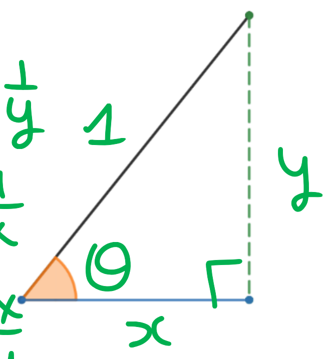


cosecant

$\sin\theta = y \rightarrow \frac{1}{\sin\theta} = \csc\theta = \frac{1}{y}$

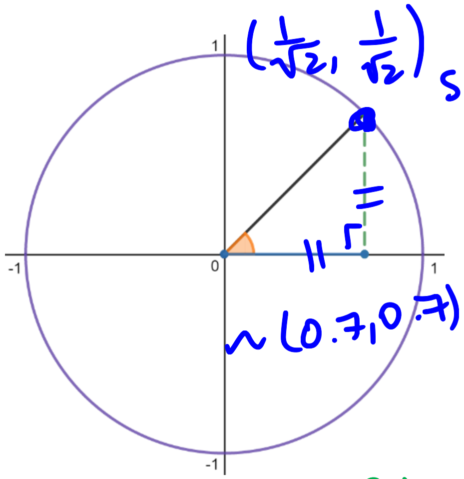
$\cos\theta = x \rightarrow \frac{1}{\cos\theta} = \sec\theta = \frac{1}{x}$

$\tan\theta = \frac{y}{x} \rightarrow \frac{1}{\tan\theta} = \cot\theta = \frac{x}{y}$



There are a few special angles notice that we build an isosceles right-angle triangle.

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$



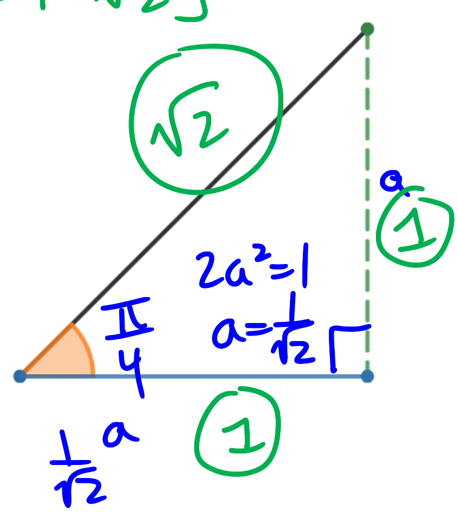
$\sim(0.7, 0.7)$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\csc\frac{\pi}{4} = \sqrt{2}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\sec\frac{\pi}{4} = \sqrt{2}$

$\tan\frac{\pi}{4} = 1$ $\cot\frac{\pi}{4} = 1$

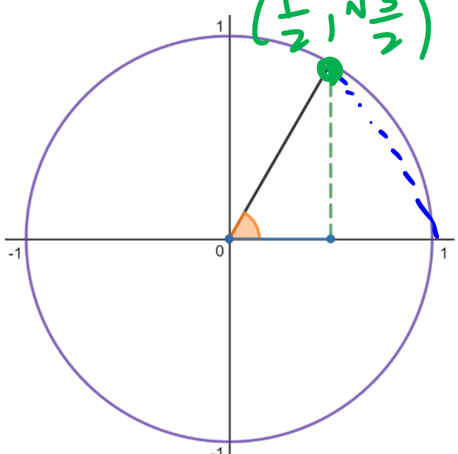
$[1 - 1 - \sqrt{2}]$



$2a^2 = 1$

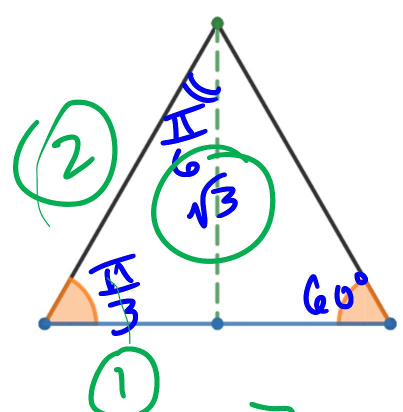
$a = \frac{1}{\sqrt{2}}$

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

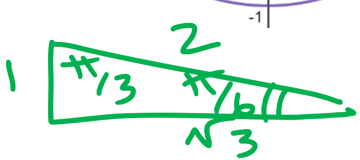


$\sin\frac{\pi}{6} = \frac{1}{2}$ $\csc\frac{\pi}{6} = 2$

$\tan\frac{\pi}{3} = \sqrt{3}$ $\cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$



$[1 - 2 - \sqrt{3}]$



Angle, θ	$\sin\theta$	$\csc\theta$	$\cos\theta$	$\sec\theta$	$\tan\theta$	$\cot\theta$
0	0	vert. asym	1	1	$\frac{0}{1} = 0$	vert. asym
$\frac{\pi}{6}$	$\frac{1}{2}$	2	$\sqrt{3}/2$	$2/\sqrt{3}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$
$\frac{\pi}{3}$						
$\frac{\pi}{2}$						
$\frac{3\pi}{4}$		see	morning			
π			notes			
3.5						
4						
4.5						
5	-0.96	-1.04	0.28	3.52	-3.38	-0.30
6						
9						

Practice Problems: 4.1 page 175 – 176 # 1-13
4.3 page 200 – 203 # 1-6, 9, 12-14, 16, 17

$\cos\theta = x$ $\sin\theta = y$ $\tan\theta = \frac{y}{x}$