The Radian and Angles

KNOW	**DO**	**UNDERSTAND**
How to recognize angles in radians. What quadrant an angle is in.	Determine coterminal angles to a given angle. Determine the trig ratios of an angle. How to use the unit circle and special triangles	*Function Characteristics*: Why the (x, y) coordinate on the unit circle is $(\cos\theta, \sin\theta)$

Vocab & Notation

- Radian
- Co-terminal
- Special Triangle
- Unit Circle
- Secant, Cosecant, Cotanget

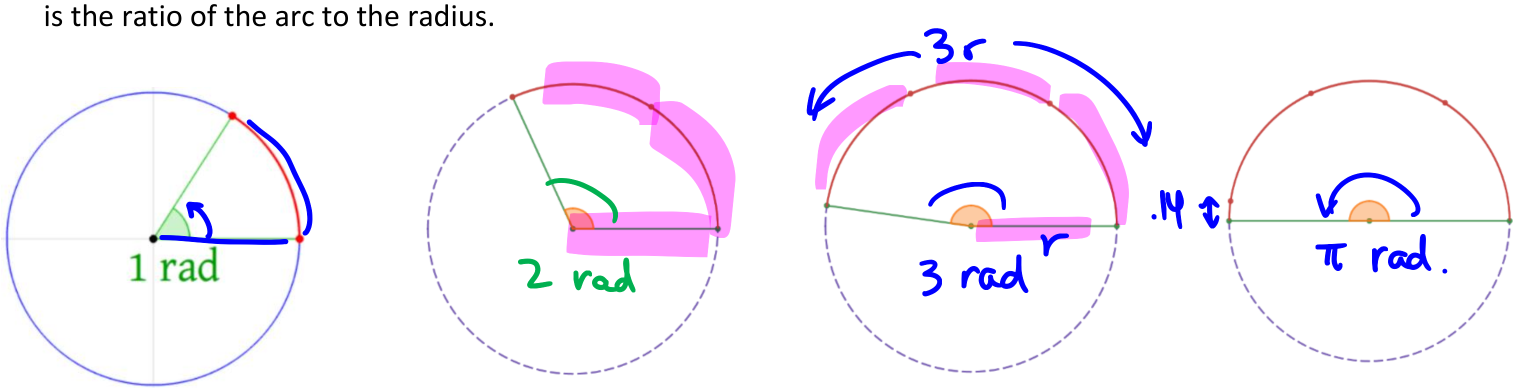
★ cosine law

Why is there 360° in a full rotation? vs 400 gradians

→ easy to divide. also ~360 days in a year

★ You think trigonometry is about triangles, but really it is about circles.

Definition: One **radian** is equal to the angle made when the arc of a circle is equal to the radius. In general, it is the ratio of the arc to the radius.



$$\text{Angle (radians)} = \frac{\text{Arc Length}}{\text{radius}} \quad \Rightarrow \quad \theta = \frac{a}{r}$$

Where θ is the angle in radians. This may look like a formula, but it is the definition of the radian. This is similar to how π is defined as the ratio between the circumference and the diameter of a circle.

If we go all around the circle, then:

arc = circum. = $2\pi r$

radius = r

$\Rightarrow$ angle $= \frac{2\pi r}{r} = 2\pi = \tau$

$= 360°$

$\pi \triangleq \frac{C}{2r}$

Example:

$\frac{\pi}{6} = \frac{1}{6}\cdot\pi \times \frac{360°}{2\pi} = 30°$

$\frac{\pi}{4} = \frac{1}{4}\pi \times \frac{180°}{\pi} = 45°$

$\frac{\pi}{2} = 90°$

$\frac{\pi}{3} = \frac{1}{3}\cdot\pi = 2\cdot\frac{1}{6}\cdot\pi = 60°$

$100° \times \frac{\pi}{180°} = \frac{5}{9}\pi$

$250° \times \frac{\pi}{180°} = \frac{25}{18}\pi = 4.36$ rad.

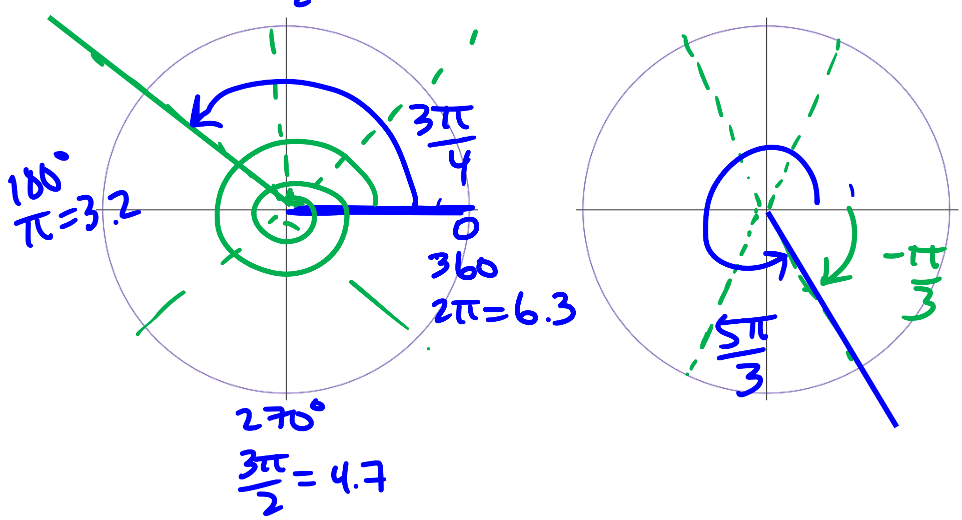
**Recall that positive angles move counter-clockwise around the circle, and negative angles move clockwise.

Definition: Angles are considered **co-terminal** if they have the same terminal arms.

Any multiple of 360° or 2π will wrap around back to the same terminal arm so we say the **general form** (which represents all the possible solutions) as

$$\theta + 2\pi n, \qquad n \in \mathbb{Z}$$

Examples:



Angle	General Form	Domain	Coterminal Angles
$\frac{19\pi}{4}$	$\frac{3\pi}{4} + 2\pi n$, $n \in \mathbb{Z}$ ($\frac{8\pi}{4}$)	$-2\pi \le \theta < 4\pi$ (3 rotations)	$\frac{3\pi}{4}$ (n=0), $\frac{11\pi}{4}$ (n=1), $-\frac{5\pi}{4}$ (n=−1)
$\frac{11\pi}{3}$	$-\frac{\pi}{3} + 2\pi n$, $n \in \mathbb{Z}$ ($6\pi/3$)	$-3\pi \le \theta < \pi$	$-\frac{\pi}{3}$ (n=0), $-\frac{7\pi}{3}$ (n=−1)
$\frac{-17\pi}{6}$	$-\frac{5\pi}{6} + 2\pi n$, $n \in \mathbb{Z}$ ($+\frac{12\pi}{6}$)	$0 \le \theta < 8\pi$ (4 rot)	$\frac{7\pi}{6}$, $\frac{19\pi}{6}$, $\frac{31\pi}{6}$, $\frac{43\pi}{6}$

soh cah toa

When defining the angle around a circle, it is useful to think of that circle on a grid centered at the origin. Such a circle is defined by the equation:

$$x^2 + y^2 = r^2$$

circle

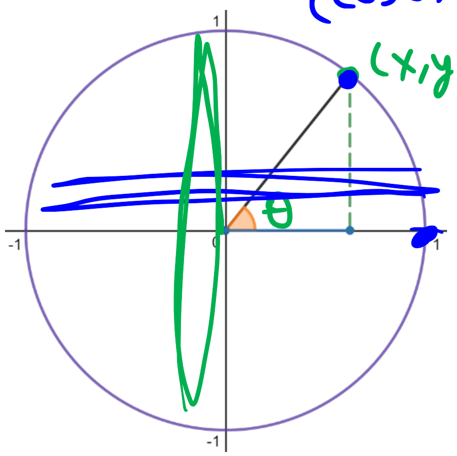
Where r is the radius of the circle.

As the angle does not change as the radius changes the **unit circle** is the circle with radius 1, centered about the origin.

$$x^2 + y^2 = 1$$

$(\cos\theta, \sin\theta)$

cosecant



$\sin\theta = y$

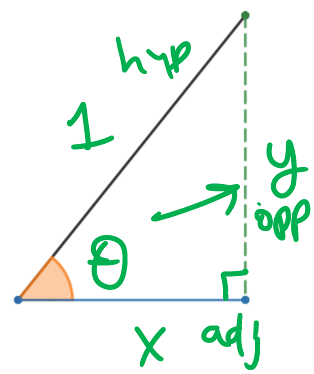
$\cos\theta = x$

$\tan\theta = \frac{y}{x}$

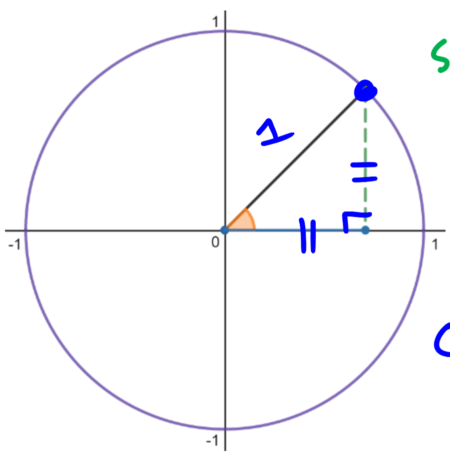
$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{y}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{x}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y}$



There are a few special angles notice that we build an isosceles right-angle triangle.



$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

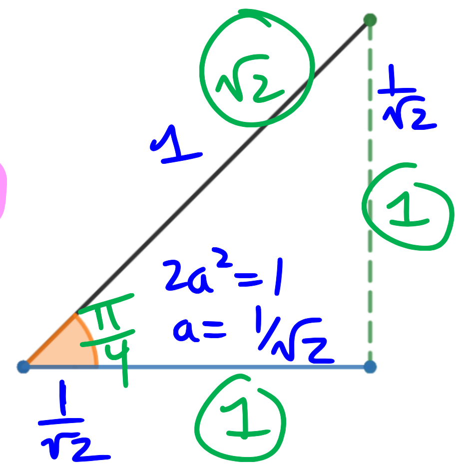
$\tan\frac{\pi}{4} = 1$

$\csc\frac{\pi}{4} = \sqrt{2}$

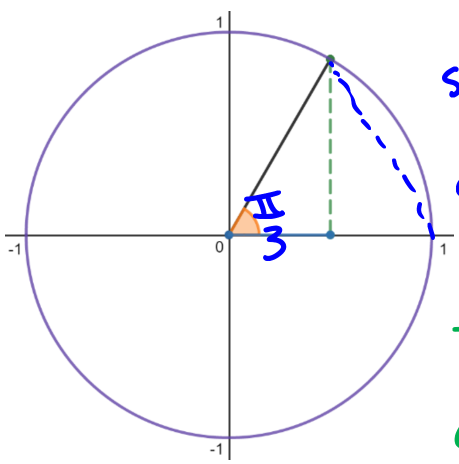
$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\sec\frac{\pi}{4} = \sqrt{2}$

$\cot\frac{\pi}{4} = 1$



$[1 - 1 - \sqrt{2}]$

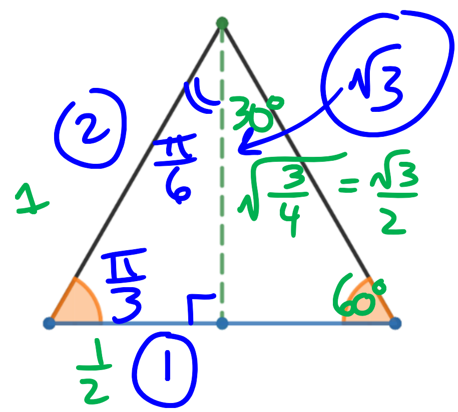


$\sin\frac{\pi}{6} = \frac{1}{2}$

$\csc\frac{\pi}{6} = 2$

$\tan\frac{\pi}{3} = \sqrt{3}$

$\cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$



$[1 - 2 - \sqrt{3}]$

$\frac{1}{\sin\theta}$

Angle, θ	$\sin\theta$	$\csc\theta$	$\cos\theta$	$\sec\theta$	$\tan\theta$	$\cot\theta$
0	0	vert. asym.	1	1	0	vert asym.
$\frac{\pi}{6}$	$\frac{1}{2}$	2	$\sqrt{3}/2$	$2/\sqrt{3}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$
$\frac{\pi}{3}$	$\sqrt{3}/2$	$\frac{2}{\sqrt{3}}$	$\frac{1}{2}$	2	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{2}$	1	1	0	vert asy	vert asy	0
$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\sqrt{2}$	$-\frac{1}{\sqrt{2}}$	$-\sqrt{2}$	-1	-1
π	0	vert asym	-1	-1	0	vert asym
3.5	-0.35	-2.85	-0.94	-1.07	0.37	2.67
4	-0.76	-1.32	-0.65	-1.53	1.16	0.86
4.5	-0.98	-1.02	-0.21	-4.74	4.64	0.22
5	-0.96	-1.04	0.28	3.52	-3.38	-0.30
6	-0.28	-3.58	0.96	1.04	-0.29	-3.44
9	0.41	2.43	-0.91	-1.10	-0.45	-2.21

Practice Problems: 4.1 page 175 – 176 # 1-13
4.3 page 200 – 203 # 1-6, 9, 12-14, 16, 17

$\cos\theta = x \qquad \sin\theta = y \qquad \tan\theta = \frac{y}{x}$