Rational Functions

KNOW	**DO**	**UNDERSTAND**
How to identify points of discontinuity, zeros, and horizontal asymptotes of a rational function.	Graph a rational function accurately. Identify the transformations that took place when working with $\frac{mx+b}{x-a}$	*Function Characteristics*: Horizontal asymptotes are not values removed from the range, but trends as $x \to \infty$. Removeable discontinuities can be filled in. *Transformations*: Can explain why horizontal and vertical stretches are equivalent in $\frac{1}{x}$

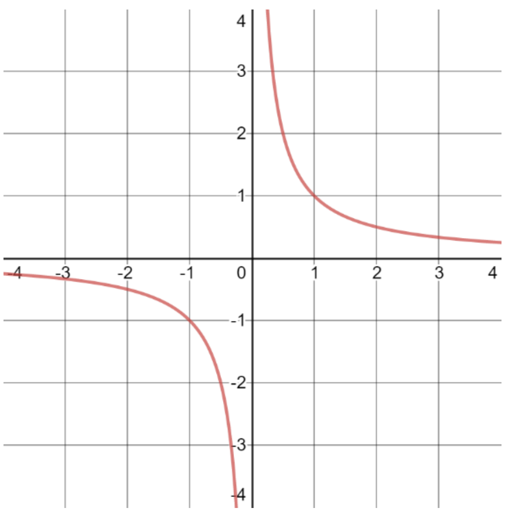
Vocab & Notation

- Discontinuity
- Removeable discontinuity

We want to consider the functions of the form:

$$\frac{p(x)}{q(x)} = \frac{ax^n + \cdots}{bx^m + \cdots}$$

To get to that point, let's consider the basic function: $f(x) = \frac{1}{x}$



Major characteristics:

If we were to transform it:

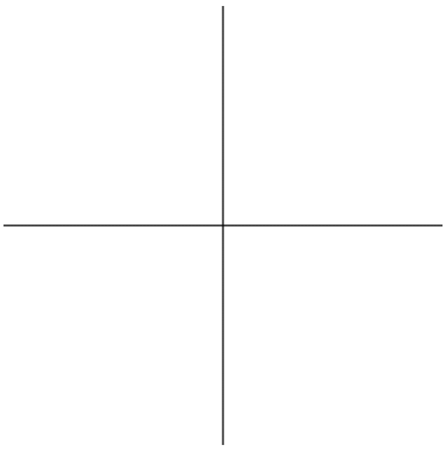
Example: Given the function $p(x) = \frac{2x}{x+2}$, identify the transformations that occurred from $\frac{1}{x}$

When we look at the rational function

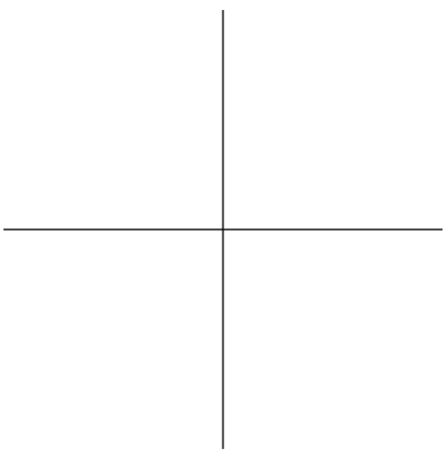
$$\frac{p(x)}{q(x)} = \frac{ax^n + \cdots}{bx^m + \cdots} = A\frac{(x-\alpha)\cdots(x-\beta)}{(x-\varphi)\cdots(x-\omega)}$$

We are going to get asymptotes (vertical and horizontal), but we are going to use the factored form to graph it when it goes beyond a degree 1 polynomial over a degree 1 polynomial.

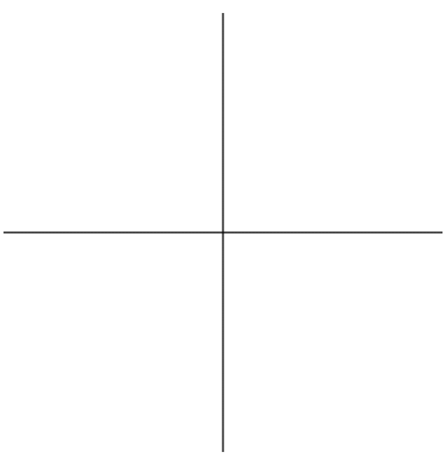
1. Discontinuities
   a. Vertical Asymptotes: $y = \frac{3}{x(x+4)^2(x-3)}$



   b. Removeable Discontinuities: $y = \frac{-4(x-3)}{(x-2)(x-3)(x+1)^2}$

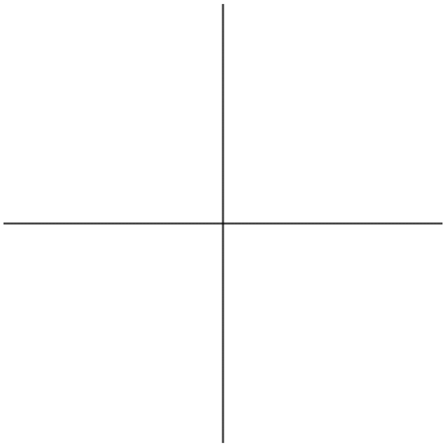


2. Zeros $y = \frac{-x(x+2)^3(x-3)^2}{x(x+6)^2(x-1)^5}$

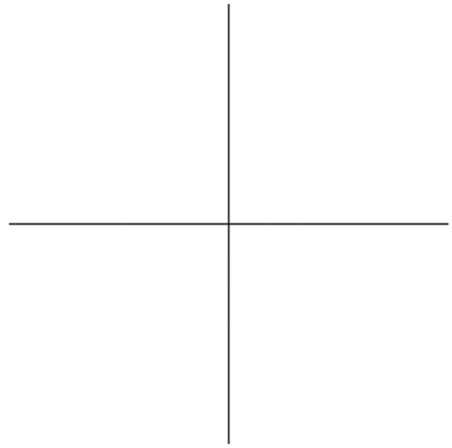


3. Horizontal asymptotes:

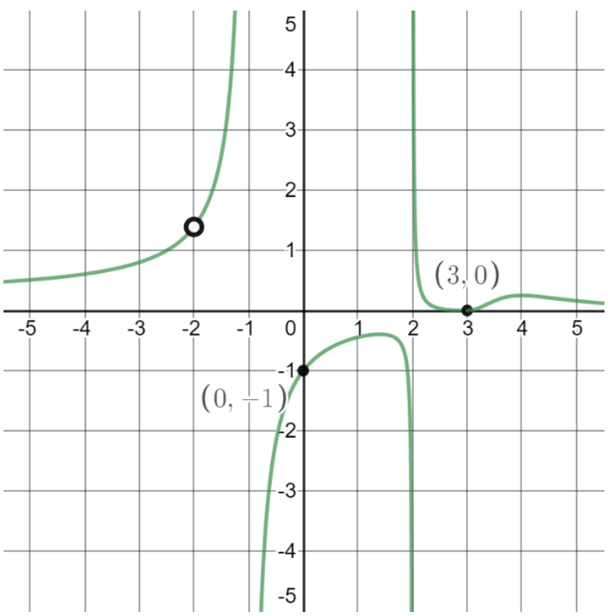
$$y = \frac{3(x+1)(x+2)}{(x-2)^2}$$

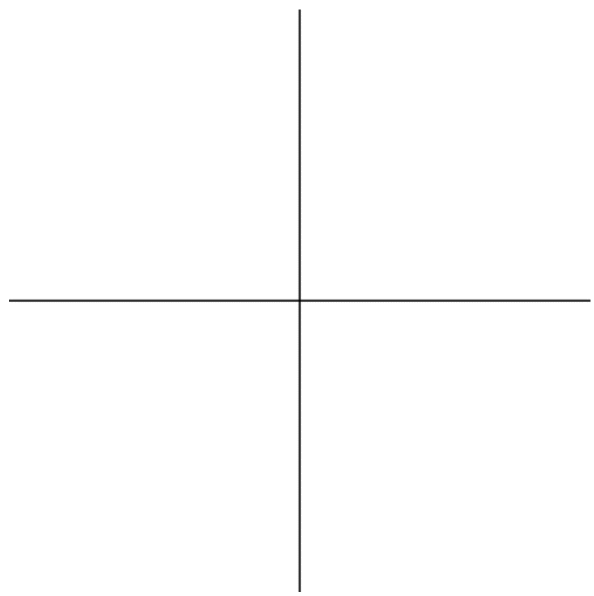


$$y = \frac{-(x+3)(x-1)^2}{(x+3)(x+2)}$$



Practice: Build an equation for the following graph and sketch the function





$$y = -\frac{2(x-3)(x-2)(x+1)}{(x-1)^2(x-3)}$$

Practice Problems: 9.1 page 442 – 445 # 1-9, 11, 12, 20-22
9.2 page 452 – 456 # 3-11, 16, 18-23, C2