Rational Functions

KNOW	**DO**	**UNDERSTAND**
How to identify points of discontinuity, zeros, and horizontal asymptotes of a rational function.	Graph a rational function accurately. Identify the transformations that took place when working with $\frac{mx+b}{x-a}$	*Function Characteristics*: Horizontal asymptotes are not values removed from the range, but trends as $x \to \infty$. Removeable discontinuities can be filled in. *Transformations*: Can explain why horizontal and vertical stretches are equivalent in $\frac{1}{x}$

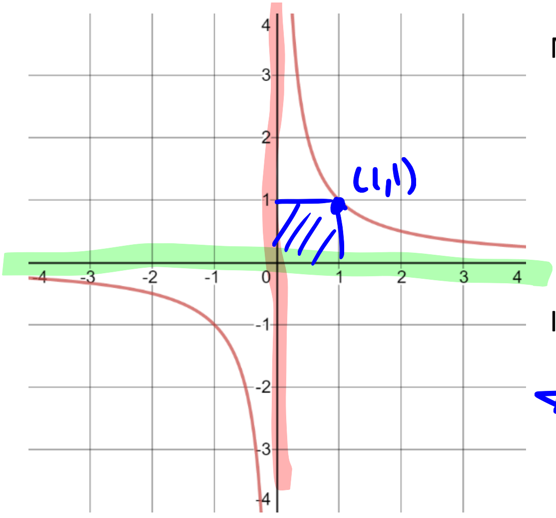
Vocab & Notation

- Discontinuity
- Removeable discontinuity

We want to consider the functions of the form:

$$\frac{p(x)}{q(x)} = \frac{ax^n + \cdots}{bx^m + \cdots}$$

To get to that point, let's consider the basic function: $f(x) = \frac{1}{x}$



Major characteristics:

vert. asymptote at $x=0$

horiz. asymptote at $y=0$

If we were to transform it:

☆ stretches + reflections don't change the asymptotes. Only shifts.

$g(x) = af(b(x-c)) + d = a \cdot \frac{1}{b(x-c)} + d = \frac{A}{x-c} + d$ ← horiz. asy.

vert. asy.

Example: Given the function $p(x) = \frac{2x}{x+2}$, identify the transformations that occurred from $\frac{1}{x}$

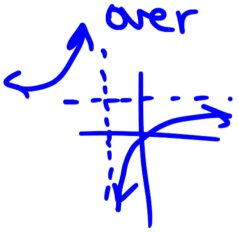
$$\begin{array}{r} 2 \\ x+2\,\overline{)\,2x} \\ -(2x+4) \\ \hline -4 \end{array}$$

$\Rightarrow \frac{2x}{x+2} = 2 - \frac{4}{x+2}$

shift up 2

left 2

vert exp by 4 and reflect over x



When we look at the rational function

$$\frac{p(x)}{q(x)} = \frac{ax^n + \cdots}{bx^m + \cdots} = A\frac{(x-\alpha)\cdots(x-\beta)}{(x-\varphi)\cdots(x-\omega)}$$

We are going to get asymptotes (vertical and horizontal), but we are going to use the factored form to graph it when it goes beyond a degree 1 polynomial over a degree 1 polynomial.

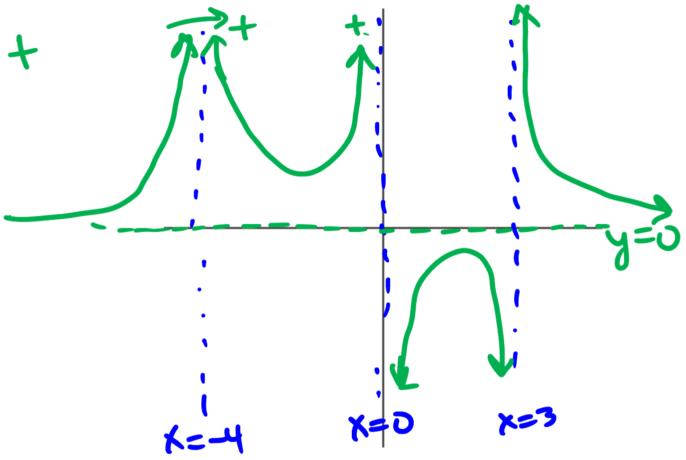
1. Discontinuities : Points not in Domain (divide by 0)
   a. Vertical Asymptotes:

$$y = \frac{3}{x(x+4)^2(x-3)}$$

x=0 mult 1

x=3 mult 1

x=-4 mult 2

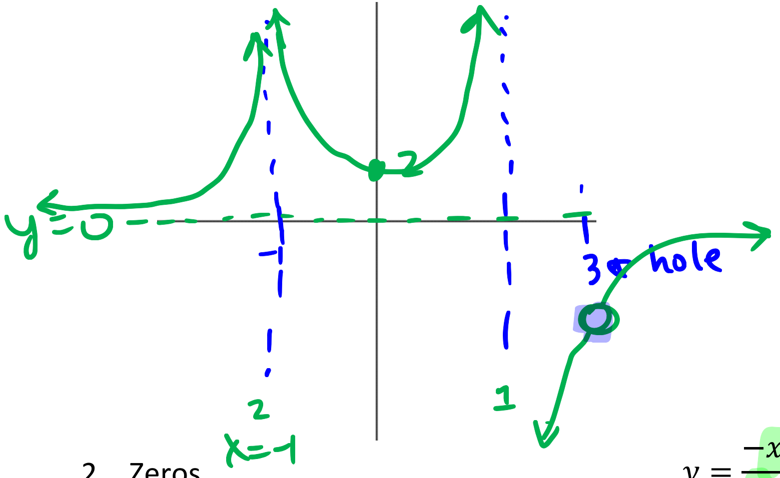


   b. Removeable Discontinuities:

$$y_1 = \frac{-4\cancel{(x-3)}}{(x-2)\cancel{(x-3)}(x+1)^2} = \frac{-4}{(x-2)(x+1)^2}, x \neq 3$$

set x=0 $y = \frac{-4}{-2\cdot 1} = 2$

$$f(x) = \begin{cases} y_1, & x \neq 3 \\ -\frac{1}{4}, & x = 3 \end{cases}$$



2. Zeros

$$y = \frac{-x(x+2)^3(x-3)^2}{x(x+6)^2(x-1)^5}$$

vert asy

x=-6 m.2

x=1 m.5

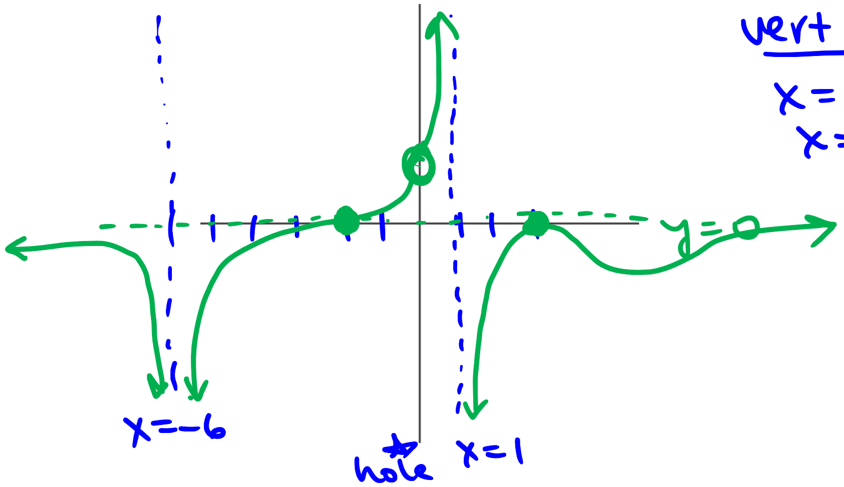
holes

x=0

zeros

x=-2 m.3

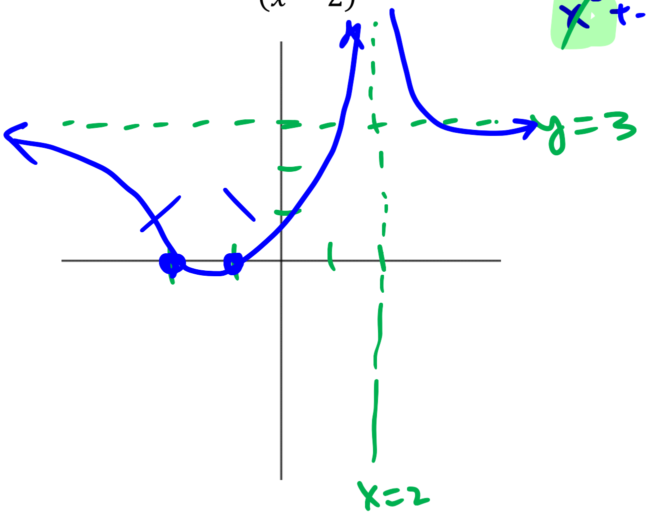
x=3 m.2



3. Horizontal asymptotes:

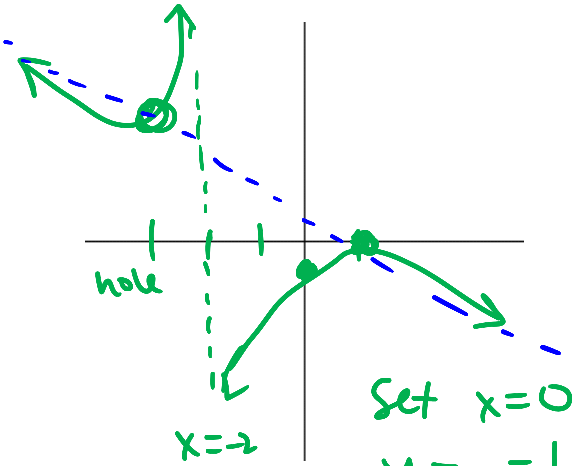
$$y = \frac{3(x+1)(x+2)}{(x-2)^2} = \frac{3\cancel{x^2}+\ldots}{\cancel{x^2}+\ldots}$$

vert asy.
x=2
holes
none
zeros
x=-1,-2



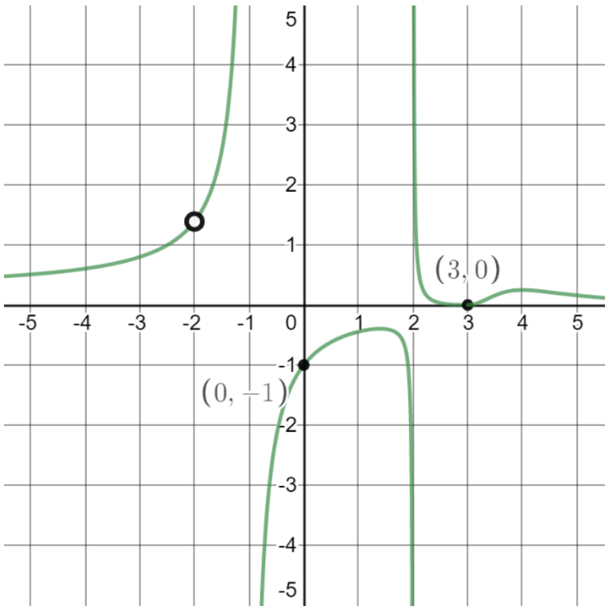
$$y = \frac{-\cancel{(x+3)}(x-1)^2}{\cancel{(x+3)}(x+2)} = \frac{-x^{\cancel{2}}+\ldots}{\cancel{x}}$$

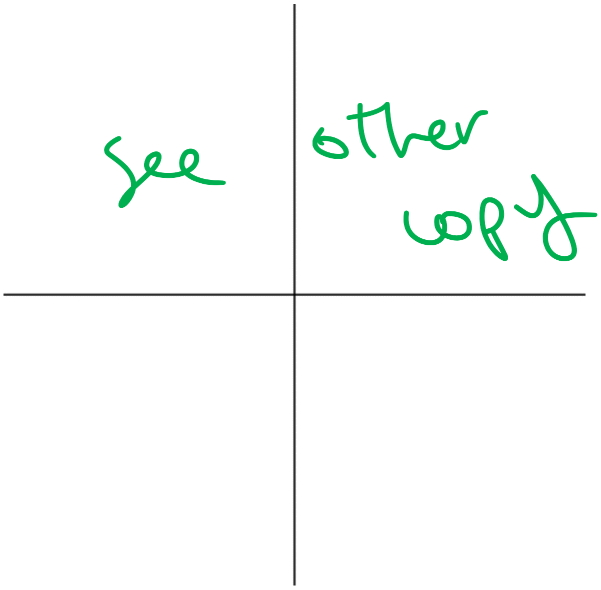
vert asy.
x=-2
holes
x=-3
zeros
x=1



Set x=0
$y = \frac{-1}{2}$

Practice: Build an equation for the following graph and sketch the function





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$$y = -\frac{2(x-3)(x-2)(x+1)}{(x-1)^2(x-3)}$$

Practice Problems: 9.1 page 442 – 445 # 1-9, 11, 12, 20-22
9.2 page 452 – 456 # 3-11, 16, 18-23, C2