Rational Functions

KNOW	**DO**	**UNDERSTAND**
How to identify points of discontinuity, zeros, and horizontal asymptotes of a rational function.	Graph a rational function accurately. Determine the equation to a rational function. Identify the transformations that took place when working with $\frac{mx+b}{x-a}$	*Function Characteristics*: Horizontal asymptotes are not values removed from the range, but trends as $x \to \infty$. Removeable discontinuities can be filled in. *Transformations*: Can explain why horizontal and vertical stretches are equivalent in $\frac{1}{x}$

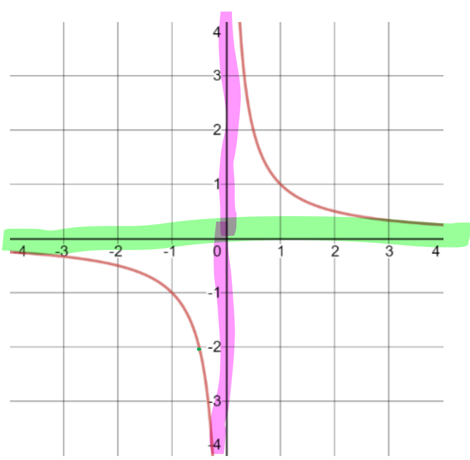
Vocab & Notation

- Discontinuity
- Removeable discontinuity

We want to consider the functions of the form:

$$\frac{p(x)}{q(x)} = \frac{ax^n + \cdots}{bx^m + \cdots}$$

To get to that point, let's consider the basic function: $f(x) = \frac{1}{x}$



Major characteristics:

vertical asymptote @ $x=0$ $\Rightarrow$ can't divide by 0

horizontal " @ $y=0$ $\Rightarrow$ as $x \to \pm\infty$ $y \to 0$

If we were to transform it:

through a shift the asymptotes will move

$$f(x-c)+d = \frac{1}{x-c} + d$$

Example: Given the function $p(x) = \frac{2x}{x+2}$, identify the transformations that occurred from $\frac{1}{x}$ $= f(x)$

$$x+2 \overline{)\,2x\,} \quad 2$$
$$-(2x+4)$$
$$-4$$

$$p(x) = 2 - \frac{4}{x+2} = -4 \cdot \frac{1}{x+2} + 2$$
$$= -4 \cdot f(x+2) + 2$$

$\Rightarrow$ vertically expanded by 4, R o x, left 2, up 2

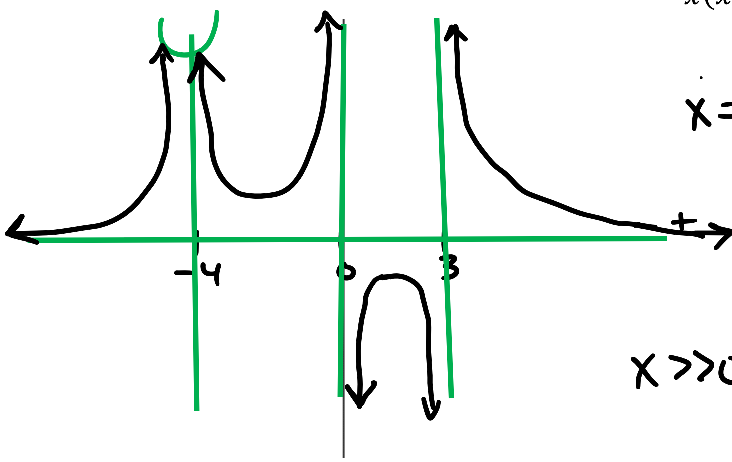
When we look at the rational function

$$\frac{p(x)}{q(x)} = \frac{ax^n + \cdots}{bx^m + \cdots} = A\frac{(x-\alpha)\cdots(x-\beta)}{(x-\varphi)\cdots(x-\omega)}$$

We are going to use the factored form to graph it when it goes beyond a degree 1 polynomial over a degree 1 polynomial.

1. Discontinuities
   a. Vertical Asymptotes:

$$y = \frac{3}{x(x+4)^2(x-3)} \sim \frac{3}{x^4}$$

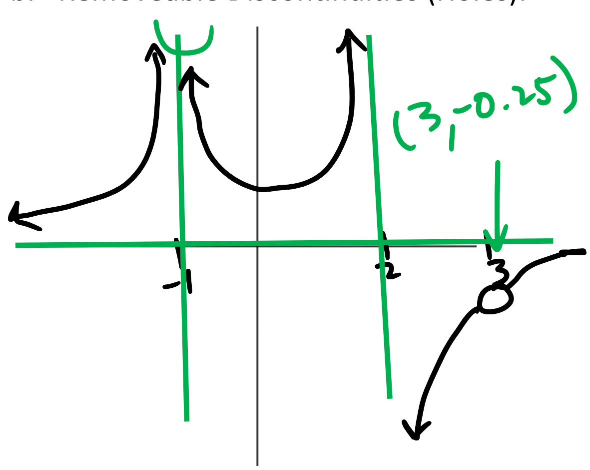


$x=0$ $\quad x=-4$ m.2 $\quad x=3$

$x \gg 0 \rightarrow y > 0$

   b. Removeable Discontinuities (Holes):

$$y = \frac{-4\cancel{(x-3)}}{(x-2)\cancel{(x-3)}(x+1)^2}, \; x \neq 3 \sim \frac{-1}{x^3}$$



vert. asym:

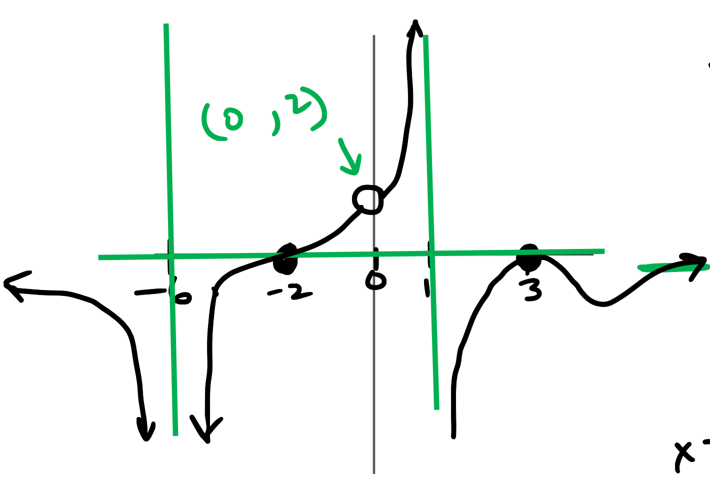
$x=2$ $\quad \cancel{x=3}$ $\quad x=-1$ m.2

Hole: $x \neq 3$

$x \gg 0 \rightarrow y < 0$

2. Zeros

$$y = \frac{-\cancel{x}(x+2)^3(x-3)^2}{\cancel{x}(x+6)^2(x-1)^5}, \; x \neq 0 \approx \frac{-x^5}{x^7} = \frac{-1}{x^2}$$



vert asym:

$x=-6$ m.2 $\quad x=1$ m.5

as $x \to \infty$

$\frac{-1}{x^2} \to 0$

holes: $x \neq 0$

zeros: $x=-2$ m.3 $\quad x=3$ m.2

$x \gg 0 \rightarrow y < 0$

3. Horizontal asymptotes:

True as $x \to \pm\infty$

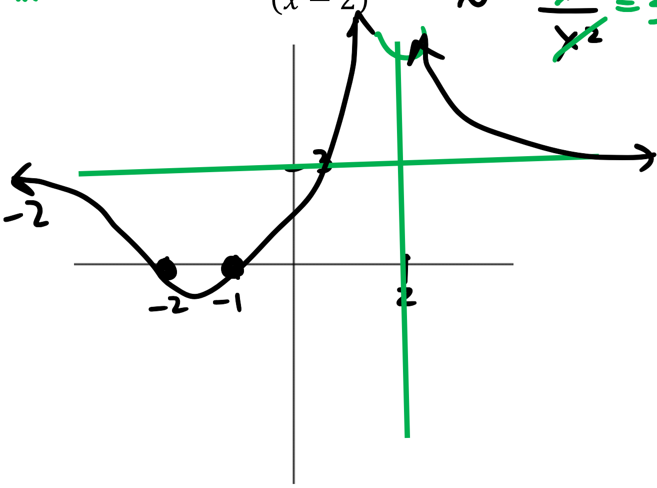
$$y = \frac{3(x+1)(x+2)}{(x-2)^2} \approx \frac{3\cancel{x^2}}{\cancel{x^2}} = 3$$

vert asym $x=2$ m.2

hole none

zero $x=-1, -2$

Horiz. asymptote $y=3$



$$y = \frac{-\cancel{(x+3)}(x-1)^2}{\cancel{(x+3)}(x+2)}$$

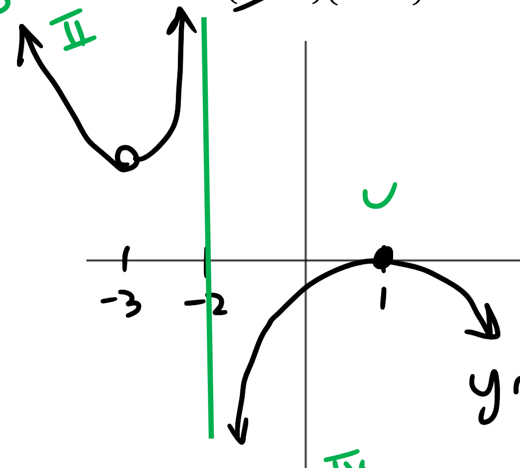
vert asym $x=-2$

hole $x \neq -3$

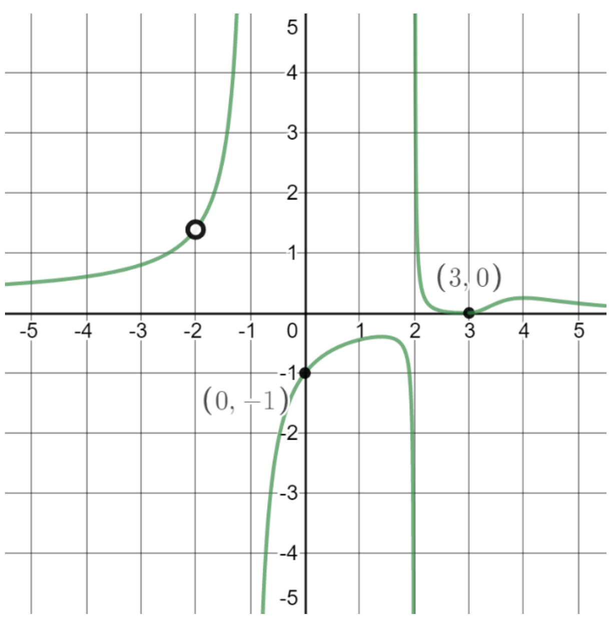
zero $x=1$ m.2

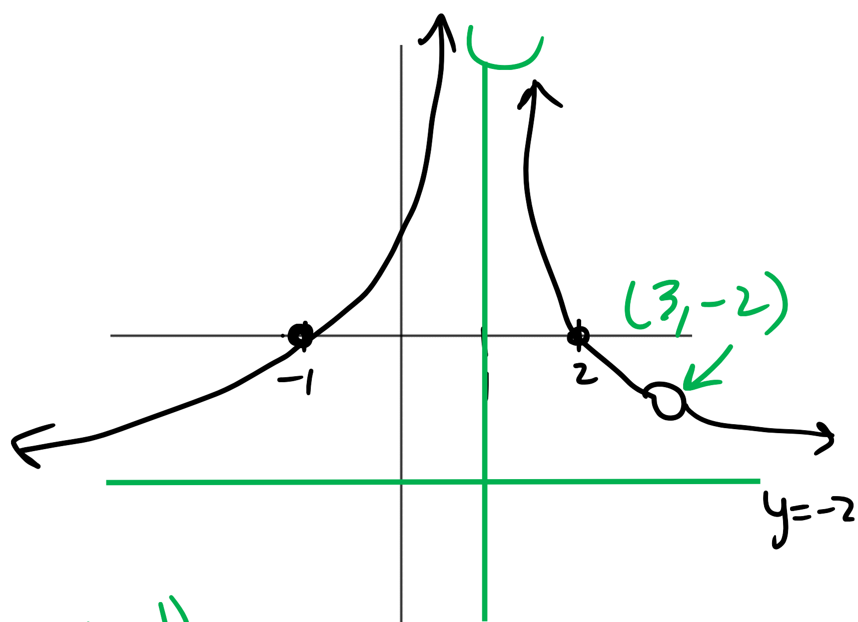
$y \sim \frac{-x^{\cancel{2}}}{\cancel{x}} = -x$

$\Rightarrow$ II to IV



Practice: Build an equation for the following graph and sketch the function





$$\frac{\cancel{(x+2)}(x-3)^2}{\cancel{(x+2)}(x+1)^3(x-2)} \cdot \frac{2}{9}$$

need so $(0,-1)$ is passed thru

need so H.A $= 0$

$$y = -\frac{2\cancel{(x-3)}(x-2)(x+1)}{(x-1)^2\cancel{(x-3)}}$$

$x \neq 3$

$$\sim -\frac{2x^2}{x^2} = -2$$

Practice Problems: 9.1 page 442 – 445 # 1-9, 11, 12, 20-22
9.2 page 452 – 456 # 3-11, 16, 18-23, C2