Rational Functions

KNOW	DO	UNDERSTAND
How to identify points of discontinuity, zeros, and horizontal asymptotes of a rational function.	Graph a rational function accurately. Determine the equation to a rational function. Identify the transformations that took place when working with $\frac{mx+b}{x-a}$	*Function Characteristics*: Horizontal asymptotes are not values removed from the range, but trends as $x \to \infty$. Removeable discontinuities can be filled in. *Transformations*: Can explain why horizontal and vertical stretches are equivalent in $\frac{1}{x}$

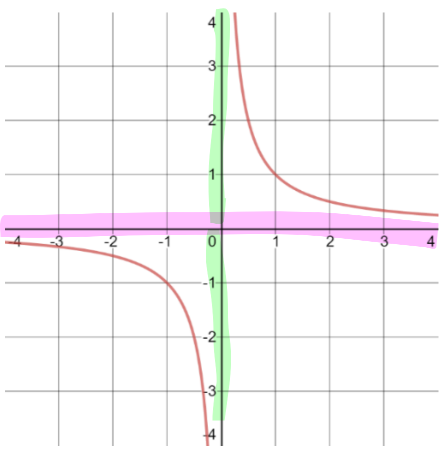
Vocab & Notation

- Discontinuity
- Removeable discontinuity

We want to consider the functions of the form:

$$\frac{p(x)}{q(x)} = \frac{ax^n + \cdots}{bx^m + \cdots}$$

To get to that point, let's consider the basic function: $f(x) = \frac{1}{x}$



Major characteristics:

Horizontal asymptote $y=0$ as $x \to \pm\infty$ $f(x) \to 0$

Vertical " $x=0 \Rightarrow f(0)$ is undefined

If we were to transform it:

the shifts will determine new asymptotes

$$f(x-c)+d = \frac{1}{x-c} + d$$

Example: Given the function $p(x) = \frac{2x}{x+2}$, identify the transformations that occurred from $\frac{1}{x} = f(x)$

$$\begin{array}{r} 2 \\ x+2 \overline{)\, 2x} \\ -(2x+4) \\ \hline -4 \end{array}$$

$$p(x) = 2 - \frac{4}{x+2} = -4 \cdot \frac{1}{x+2} + 2$$

$$= -4\, f(x+2) + 2$$

$\Rightarrow$ vertical expansion by 4, RoX, left 2, up 2

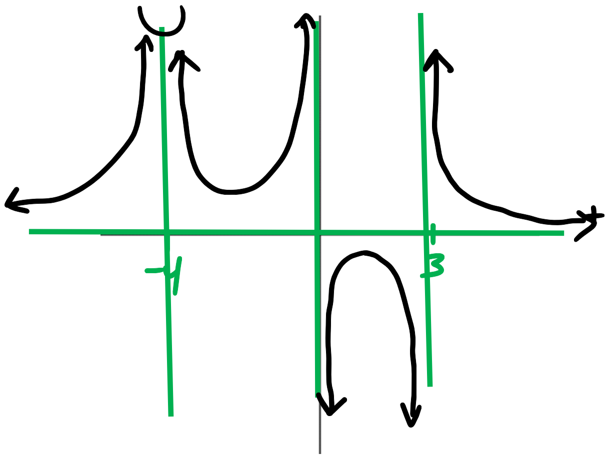
When we look at the rational function

$$\frac{p(x)}{q(x)} = \frac{ax^n + \cdots}{bx^m + \cdots} = A\frac{(x-\alpha)\cdots(x-\beta)}{(x-\varphi)\cdots(x-\omega)}$$

We are going to use the factored form to graph it when it goes beyond a degree 1 polynomial over a degree 1 polynomial.

1. Discontinuities
   a. Vertical Asymptotes:

$$y = \frac{3}{x(x+4)^2(x-3)}$$



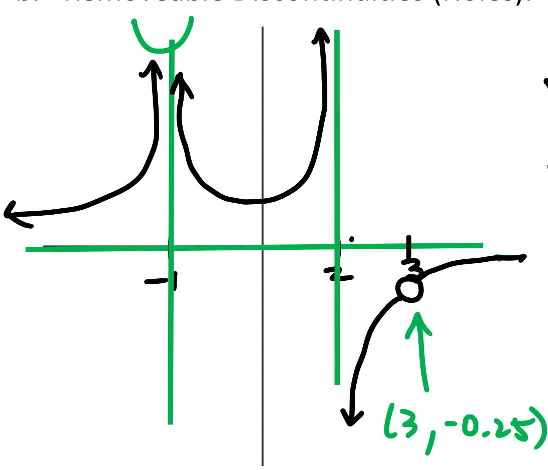
we can't divide by 0

$\Rightarrow$ $x=0$, $x=-4$ *m.2, $x=3$

for $x \gg 0$ $y>0$

   b. Removeable Discontinuities (Holes):

$$y = \frac{-4\cancel{(x-3)}}{(x-2)\cancel{(x-3)}(x+1)^2}, \quad x \neq 3$$



vert. asymptotes $x=2$, ~~x=3~~, $x=-1$ *m 2

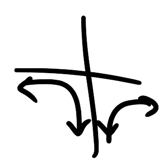
horiz. asymptote $y=0$

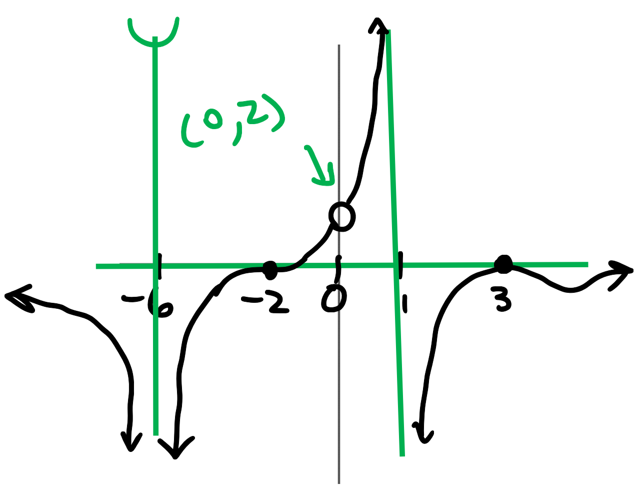
Hole @ $x=3$

$x \gg 0 \rightarrow y<0$

2. Zeros

$$y = \frac{-\cancel{x}(x+2)^3(x-3)^2}{\cancel{x}(x+6)^2(x-1)^5}, \quad x \neq 0 \quad \sim \frac{-x^5}{x^7} = \frac{-1}{x^2}$$





horiz. asymptote $y=0$

vert. " $x=-6$ m. 2 $x=1$ m. 5

holes $x \neq 0$

zeros $x=-2$ m.3 $x=3$ m. 2

$x \gg 0 \rightarrow y<0$

True for $x >> 0$

3. Horizontal asymptotes:

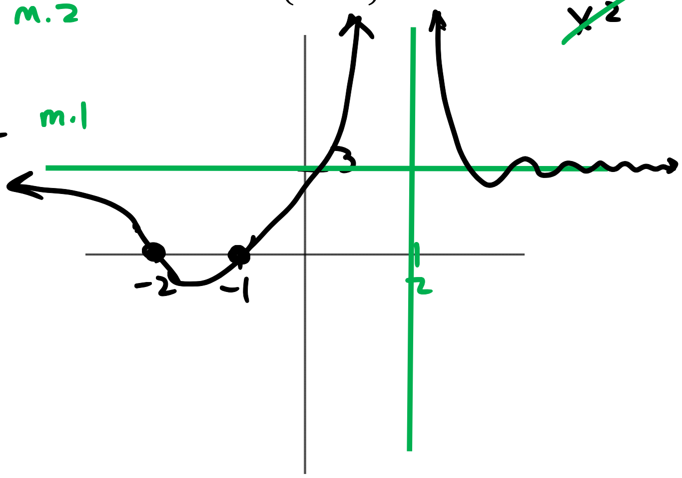
vert asy: $x=2$ m.2

zero: $x=-1, -2$ m.1

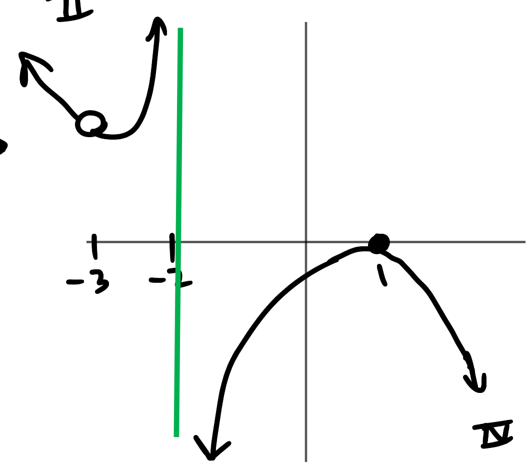
hole: None

horiz asy: $y=3$

$$y = \frac{3(x+1)(x+2)}{(x-2)^2} \sim \frac{3\cancel{x^2}}{\cancel{x^2}}$$



$$y = \frac{-\cancel{(x+3)}(x-1)^2}{\cancel{(x+3)}(x+2)} \sim \frac{-x^2}{x} = -x$$



vert asym $x=-2$ M.1

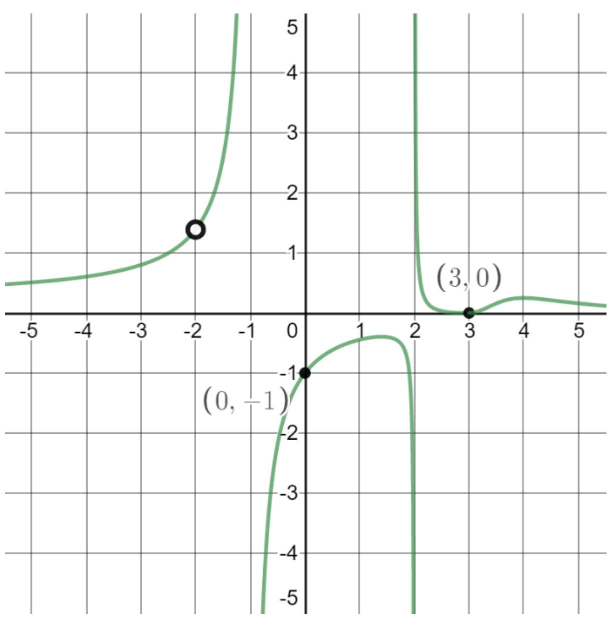
zero $x=1$ m.2

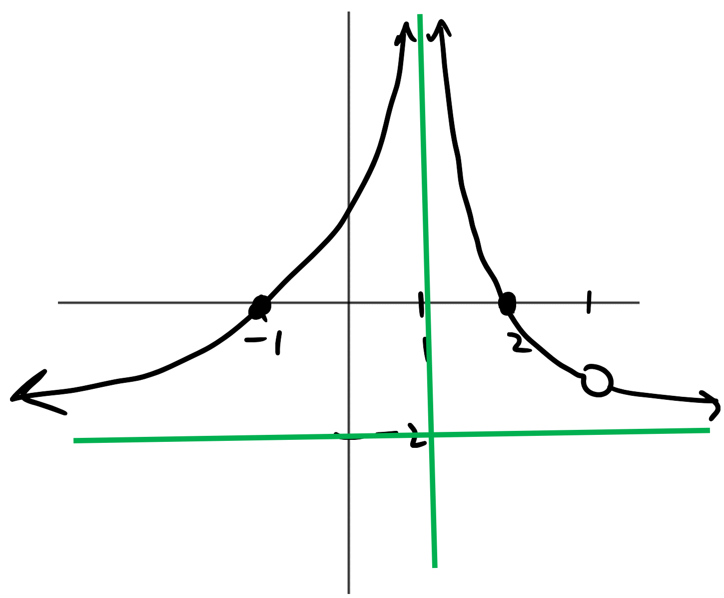
hole $x \neq -3$

horiz. asy: None

II → IV

Practice: Build an equation for the following graph and sketch the function





$$\frac{(x-3)^2(x+2)}{(x-2)(x+1)^3(x+2)} \cdot \frac{2}{9}$$

$$-\frac{2\cancel{x^2}}{\cancel{x^2}} \sim y = -\frac{2\cancel{(x-3)}(x-2)(x+1)}{(x-1)^2\cancel{(x-3)}}$$

$x \neq 3$ hole

zero $x = 2, -1$

vert asym: $x=1$ m.2

Practice Problems: 9.1 page 442 – 445 # 1-9, 11, 12, 20-22
9.2 page 452 – 456 # 3-11, 16, 18-23, C2