Rational Models Practice

Goal: Be able to model rational functions in general cases and interpret the model in a meaningful way.

Scenario 1 (with time):

A. In situation A, I imagine two taps that work together to fill a bathtub that can hold 100L of water. Tap B can fill up 6L two times faster than tap A. A person turns on tap B first. After 3 minutes she then turns on tap A.

What does the efficiency of tap B need to be to fill up the 100L bathtub in the next 3 minutes?

Tap A can fill up 6L in t minutes.
↳ Tap B can fill up 6L in $2t$ minutes.

total time $\rightarrow$ $$\boxed{3} = \frac{\boxed{100 - 3 \cdot \frac{\cancel{6}^3}{\cancel{2}t}}}{\boxed{\frac{6}{t} + \frac{\cancel{6}^3}{\cancel{2}t}}}$$ ← volume two taps fill together ← total efficiency of the taps working together

$$3 = \frac{100 - \frac{9}{t}}{\frac{9}{t}}$$
$$= \frac{t}{9}\left(100 - \frac{9}{t}\right)$$
$$3 = \frac{100}{9}t - 1$$
$$\frac{100}{9}t = 4$$
$$t = 0.36 \text{ minutes}$$

$\therefore$ efficiency of tap $B = \frac{6}{2 \times 0.36} = \frac{25}{3} = 8.33$ L/minutes

B. In situation B, I imagine two types of plants are in a room, 20 plants of type A and 40 plants of type B. A plant A can produce 1L of oxygen 30 minutes faster than a plant B.

After 4 hours, the total amount of oxygen produced working together is equal to the oxygen produced with 50 plants of type A working alone, what is the efficiency of oxygen production for plants A and B at that time?

A plant B can produce 1 L of oxygen in t minutes.
A plant A can produce 1 L of oxygen in $(t-30)$ minutes.
$\Rightarrow$ efficiency of plant $B = \frac{1}{t}$
efficiency of plant $A = \frac{1}{t-30}$

4 hrs $= 4 \times 60 = 240$ mins

time → ← # of plant A ← # of plant B ← time ← # of plant A alone

$$\boxed{\cancel{240}}\left(\frac{\boxed{20}}{t-30} + \frac{\boxed{40}}{t}\right) = \boxed{\cancel{240}} \cdot \frac{\boxed{50}}{t-30}$$

$$20t + 40(t-30) = 50t$$
$$20t + 40t - 1200 = 50t$$
$$60t - 1200 = 50t$$
$$t = \frac{1200}{10}$$
$$t = 120 \text{ minutes}$$

efficiency of plant $B = \frac{1}{120} = 0.0083$ L/min
efficiency of plant $A = \frac{1}{120-30} = 0.011$ L/min

* Questions to ask yourself:
1. Why the time, 4 hrs does not matter here?
2. What will happen to t if the # of plant A working alone is 80?

Ans: https://www.desmos.com/calculator/4km8ovusdb

Scenario 2 (**without** time):

C. In situation C, I imagine two types of mobile plans are available. Bell costs 0.5 dollars per call and Tellus costs 0.8 dollars per call. We make 9 calls with Bell and 14 calls with Tellus.

What the overall cost per call is?

$$C = \frac{9 \times 0.5 + 14 \times 0.8}{9+14} = 0.683 \$/\text{call}$$

D. In situation D, I imagine, in a local household, heater A costs \$5.00 to raise the room temperature by 10°C. A more efficient heater, heater B, costs \$3.00 to raise the same room by 10°C.

When 1 heater of type A, and several heaters of type B works separately in rooms of the same size, how many B heater are needed to control the price at \$3.50 per 10°C?

Let n = # of heater B.

$$\frac{1 \cdot 5 + n \cdot 3}{1+n} = 3.5$$

$$\frac{5+3n}{1+n} = 3.5$$

$$5 + 3n = 3.5 + 3.5n$$

$$0.5n = 1.5$$

$$n = 3$$

$\therefore$ 3 B heaters are needed