

Discuss the Ideas

- When a is a rational number and n is a natural number, what does $a^{\frac{1}{n}}$ represent?
- When a is a rational number and m and n are natural numbers, what does $a^{\frac{m}{n}}$ represent?

Exercises

A

3. Evaluate each power without using a calculator.

a) $16^{\frac{1}{2}}$ b) $36^{\frac{1}{2}}$ c) $64^{\frac{1}{3}}$
 d) $32^{\frac{1}{5}}$ e) $(-27)^{\frac{1}{3}}$ f) $(-1000)^{\frac{1}{3}}$

4. Evaluate each power without using a calculator.

a) $100^{0.5}$ b) $81^{0.25}$
 c) $1024^{0.2}$ d) $(-32)^{0.2}$

5. Write each power as a radical.

a) $36^{\frac{1}{3}}$ b) $48^{\frac{1}{2}}$ c) $(-30)^{\frac{1}{5}}$

6. Write each radical as a power.

a) $\sqrt{39}$ b) $\sqrt[4]{90}$
 c) $\sqrt[3]{29}$ d) $\sqrt[5]{100}$

7. Evaluate each power without using a calculator.

a) 8^0 b) $8^{\frac{1}{3}}$ c) $8^{\frac{2}{3}}$
 d) $8^{\frac{3}{3}}$ e) $8^{\frac{4}{3}}$ f) $8^{\frac{5}{3}}$

B

8. Write each power as a radical.

a) $4^{\frac{2}{3}}$ b) $(-10)^{\frac{3}{5}}$ c) $2.3^{\frac{3}{2}}$

9. A cube has a volume of 350 cm^3 . Write the edge length of the cube as a radical and as a power.

10. Write each power as a radical.

a) $48^{\frac{2}{3}}$ b) $(-1.8)^{\frac{5}{3}}$ c) $\left(\frac{3}{8}\right)^{2.5}$
 d) $0.75^{0.75}$ e) $\left(-\frac{5}{9}\right)^{\frac{2}{5}}$ f) $1.25^{1.5}$

11. Write each radical as a power.

a) $\sqrt{3.8^3}$ b) $(\sqrt[3]{-1.5})^2$ c) $\sqrt[4]{\left(\frac{9}{5}\right)^5}$
 d) $\sqrt[3]{\left(\frac{3}{8}\right)^4}$ e) $\left(\sqrt{\frac{5}{4}}\right)^3$ f) $\sqrt[5]{(-2.5)^3}$

12. Evaluate each power without using a calculator.

a) $9^{\frac{3}{2}}$ b) $\left(\frac{27}{8}\right)^{\frac{2}{3}}$ c) $(-27)^{\frac{2}{3}}$
 d) $0.36^{1.5}$ e) $(-64)^{\frac{2}{3}}$ f) $\left(\frac{4}{25}\right)^{\frac{3}{2}}$

13. Write an equivalent form for each number using a power with exponent $\frac{1}{2}$, then write the answer as a radical.

a) 2 b) 4 c) 10 d) 3 e) 5

14. Write an equivalent form for each number using a power with exponent $\frac{1}{3}$, then write the answer as a radical.

a) -1 b) 2 c) 3 d) -4 e) 4

15. Arrange these numbers in order from least to greatest. Describe your strategy.

$\sqrt[3]{4}, 4^{\frac{3}{2}}, 4^2, \left(\frac{1}{4}\right)^{\frac{3}{2}}$

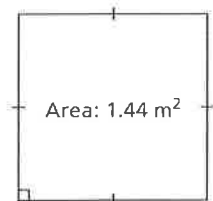
16. a) Evaluate.

i) $16^{1.5}$ ii) $81^{0.75}$
 iii) $(-32)^{0.8}$ iv) $35^{0.5}$
 v) $1.21^{1.5}$ vi) $\left(\frac{3}{4}\right)^{0.6}$

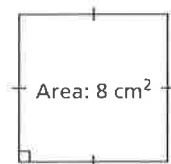
b) Which powers in part a could you have evaluated without a calculator? How can you tell before you evaluate?

8. Areas of squares may vary. For example:

a)



b)



9. a) $3\sqrt{5}$ b) $2\sqrt[3]{12}$
 c) Cannot be simplified d) $2\sqrt[4]{3}$
 e) $2\sqrt[3]{10}$ f) Cannot be simplified
11. a) $\sqrt{63}$ b) $\sqrt[3]{32}$
 c) $\sqrt{147}$ d) $\sqrt[4]{192}$
 e) $\sqrt[3]{270}$ f) $\sqrt{396}$

4.4 Fractional Exponents and Radicals, page 227

3. a) 4 b) 6
 c) 4 d) 2
 e) -3 f) -10
4. a) 10 b) 3
 c) 4 d) -2
5. a) $\sqrt[3]{36}$ b) $\sqrt{48}$
 c) $\sqrt[3]{-30}$
6. a) $39^{\frac{1}{2}}$ b) $90^{\frac{1}{4}}$
 c) $29^{\frac{1}{3}}$ d) $100^{\frac{1}{5}}$
7. a) 1 b) 2
 c) 4 d) 8
 e) 16 f) 32
8. a) $\sqrt[3]{4^2}$, or $(\sqrt[3]{4})^2$
 b) $\sqrt[3]{(-10)^3}$, or $(\sqrt[3]{-10})^3$
 c) $\sqrt{2.3^3}$, or $(\sqrt{2.3})^3$
9. $\sqrt[3]{350}$ cm, $350^{\frac{1}{3}}$ cm

10. a) $\sqrt[3]{48^2}$, or $(\sqrt[3]{48})^2$
 b) $\sqrt[3]{(-1.8)^4}$, or $(\sqrt[3]{-1.8})^4$
 c) $\sqrt{\left(\frac{3}{8}\right)^5}$, or $\left(\sqrt{\frac{3}{8}}\right)^5$
 d) $\sqrt[4]{0.75^3}$, or $(\sqrt[4]{0.75})^3$
 e) $\sqrt[3]{\left(-\frac{5}{9}\right)^2}$, or $\left(\sqrt[3]{-\frac{5}{9}}\right)^2$
 f) $\sqrt{1.25^3}$, or $(\sqrt{1.25})^3$
11. a) $3.8^{\frac{3}{2}}$, or $3.8^{1.5}$ b) $(-1.5)^{\frac{2}{3}}$
 c) $\left(\frac{9}{5}\right)^{\frac{5}{4}}$, or $\left(\frac{9}{5}\right)^{1.25}$ d) $\left(\frac{3}{8}\right)^{\frac{4}{3}}$
 e) $\left(\frac{5}{4}\right)^{\frac{3}{2}}$, or $\left(\frac{5}{4}\right)^{1.5}$ f) $(-2.5)^{\frac{3}{5}}$, or $(-2.5)^{0.6}$
12. a) 27 b) $\frac{9}{4}$
 c) 9 d) 0.216
 e) 16 f) $\frac{8}{125}$
13. a) $4^{\frac{1}{2}}$, $\sqrt{4}$ b) $16^{\frac{1}{2}}$, $\sqrt{16}$
 c) $100^{\frac{1}{2}}$, $\sqrt{100}$ d) $9^{\frac{1}{2}}$, $\sqrt{9}$
 e) $25^{\frac{1}{2}}$, $\sqrt{25}$
14. a) $(-1)^{\frac{1}{3}}$, $\sqrt[3]{-1}$ b) $8^{\frac{1}{3}}$, $\sqrt[3]{8}$
 c) $27^{\frac{1}{3}}$, $\sqrt[3]{27}$ d) $(-64)^{\frac{1}{3}}$, $\sqrt[3]{-64}$
 e) $64^{\frac{1}{3}}$, $\sqrt[3]{64}$
15. $\left(\frac{1}{4}\right)^{\frac{3}{2}}$, $\sqrt[3]{4}$, $4^{\frac{3}{2}}$, 4^2
16. a) i) 64 ii) 27
 iii) 16 iv) 5.9160...
 v) 1.331 v) 0.8414...
 b) i, ii, iii, v
17. Approximately 76 m
 18. 2.744
 19. Approximately 1.3 m²

7. Write as a single power.

a) $\left[\left(\frac{3}{5}\right)^3\right]^4$

b) $\left[\left(\frac{3}{5}\right)^3\right]^{-4}$

c) $\left[\left(\frac{3}{5}\right)^{-3}\right]^{-4}$

d) $\left[\left(-\frac{3}{5}\right)^{-3}\right]^{-4}$

8. Simplify.

a) $\left(\frac{a}{b}\right)^2$

b) $\left(\frac{n^2}{m}\right)^3$

c) $\left(\frac{c^2}{d^2}\right)^{-4}$

d) $\left(\frac{2b}{5c}\right)^2$

e) $(ab)^2$

f) $(n^2m)^3$

g) $(c^3d^2)^{-4}$

h) $(xy^{-1})^3$

B

9. Simplify. State the exponent law you used.

a) $x^{-3} \cdot x^4$

b) $a^{-4} \cdot a^{-1}$

c) $b^4 \cdot b^{-3} \cdot b^2$

d) $m^8 \cdot m^{-2} \cdot m^{-6}$

e) $\frac{x^{-5}}{x^2}$

f) $\frac{s^5}{s^{-5}}$

g) $\frac{b^{-8}}{b^{-3}}$

h) $\frac{t^{-4}}{t^{-4}}$

10. Evaluate.

a) $1.5^{\frac{3}{2}} \cdot 1.5^{\frac{1}{2}}$

b) $\left(\frac{3}{4}\right)^{\frac{3}{4}} \cdot \left(\frac{3}{4}\right)^{\frac{5}{4}}$

c) $(-0.6)^{\frac{1}{3}} \cdot (-0.6)^{\frac{5}{3}}$

d) $\left(\frac{4}{5}\right)^{\frac{4}{3}} \cdot \left(\frac{4}{5}\right)^{\frac{4}{3}}$

e) $\frac{0.6^{\frac{1}{2}}}{0.6^{\frac{3}{2}}}$

f) $\frac{\left(-\frac{3}{8}\right)^{\frac{2}{3}}}{\left(-\frac{3}{8}\right)^{\frac{1}{3}}}$

g) $\frac{0.49^{\frac{5}{2}}}{0.49^4}$

h) $\frac{0.027^{\frac{5}{3}}}{0.027^{\frac{4}{3}}}$

11. Simplify. Explain your reasoning.

a) $(x^{-1}y^{-2})^{-3}$

b) $(2a^{-2}b^2)^{-2}$

c) $(4m^2n^3)^{-3}$

d) $\left(\frac{3}{2}m^{-2}n^{-3}\right)^{-4}$

12. A cone with equal height and radius has volume 1234 cm^3 . What is the height of the cone to the nearest tenth of a centimetre?

13. A sphere has volume 375 cubic feet. What is the surface area of the sphere to the nearest square foot?

14. Simplify. Which exponent laws did you use?

a) $\frac{(a^2b^{-1})^{-2}}{(a^{-3}b)^3}$

b) $\left(\frac{(c^{-3}d)^{-1}}{c^2d}\right)^{-2}$

15. Evaluate each expression for $a = -2$ and $b = 1$. Explain your strategy.

a) $(a^3b^2)(a^2b^3)$

b) $(a^{-1}b^{-2})(a^{-2}b^{-3})$

c) $\frac{a^{-4}b^5}{ab^3}$

d) $\left(\frac{a^{-7}b^7}{a^{-9}b^{10}}\right)^{-5}$

16. Simplify.

a) $m^{\frac{2}{3}} \cdot m^{\frac{4}{3}}$

b) $x^{\frac{3}{2}} \div x^{\frac{1}{4}}$

c) $\frac{-9a^{-4}b^4}{3a^2b^{\frac{1}{4}}}$

d) $\left(\frac{-64c^6}{a^9b^{\frac{1}{2}}}\right)^{\frac{1}{3}}$

17. Identify any errors in each solution for simplifying an expression. Write a correct solution.

a) $(x^2y^{-3})(x^{\frac{1}{2}}y^{-1}) = x^2 \cdot x^{\frac{1}{2}} \cdot y^{-3} \cdot y^{-1}$
 $= x^{\frac{5}{2}} \cdot y^{-4}$
 $= xy^3$

b) $\left(\frac{-5a^2}{b^{\frac{1}{2}}}\right)^{-2} = \frac{10a^{-4}}{b^{-1}}$
 $= \frac{10b}{a^4}$

18. Explain how to use a measuring cylinder containing water to calculate the diameter of a marble that fits inside the cylinder.

19. Identify the errors in each simplification. Write the correct solution.

a) $\frac{(m^{-3} \cdot n^2)^{-4}}{(m^2 \cdot n^{-3})^2} = (m^{-5} \cdot n^5)^{-6}$
 $= m^{30} \cdot n^{30}$
 $= (mn)^{30}$

b) $\left(\frac{1}{r^2} \cdot s^{\frac{3}{2}}\right)^{\frac{1}{2}} \cdot \left(r^{\frac{1}{4}} \cdot s^{\frac{1}{2}}\right)^{-1} = r^1 \cdot s^{-1} \cdot r^{\frac{5}{4}} \cdot s^{\frac{1}{2}}$
 $= r^{1+\frac{5}{4}} \cdot s^{-1+\frac{1}{2}}$
 $= r^{\frac{9}{4}} \cdot s^{-\frac{1}{2}}$
 $= \frac{1}{r^{\frac{9}{4}} \cdot s^{\frac{1}{2}}}$

2. a) i) $\sqrt[3]{35^2}$, or $(\sqrt[3]{35})^2$
 ii) $\sqrt{32^3}$, or $(\sqrt{32})^3$
 iii) $\sqrt[3]{(-32)^2}$, or $(\sqrt[3]{-32})^2$
 iv) $\sqrt{400^3}$, or $(\sqrt{400})^3$
 v) $\sqrt[3]{-125}$
 vi) $\sqrt[3]{\left(\frac{8}{125}\right)^2}$, or $\left(\sqrt[3]{\frac{8}{125}}\right)^2$
- b) iii) 4 iv) 8000
 v) -5 vi) $\frac{4}{25}$

3. a) $4^{\frac{1}{3}}$
 b) $9^{\frac{1}{2}}$, or $9^{0.5}$
 c) $18^{\frac{1}{4}}$, or $18^{0.25}$
 d) $10^{\frac{3}{2}}$, or $10^{1.5}$
 e) $(-10)^{\frac{2}{3}}$

4. Approximately 53 s
 5. $\sqrt[3]{3}$, $3^{\frac{2}{3}}$, $(\sqrt[3]{3})^4$, $3^{\frac{3}{2}}$, $(\sqrt{3})^5$
 6. $\sqrt[3]{421\,875}$ mm, $421\,875^{\frac{1}{3}}$ mm, 75 mm

7. a) $\frac{81}{16}$ b) 4
 c) $\frac{1}{100}$ d) 2
 e) 100 f) 625
8. \$4589.06

4.6 Applying the Exponent Laws, page 241

3. a) x^7 b) $\frac{1}{a^3}$
 c) b^2 d) $\frac{1}{m}$
4. a) 0.5^5 b) 0.5^{-1}
 c) 0.5^{-1} d) 0.5^5
5. a) x^{-2} b) $\frac{1}{x^3}$
 c) n d) $\frac{1}{a^4}$

6. a) n^6 b) $\frac{1}{z^6}$
 c) n^{12} d) $\frac{1}{c^4}$
7. a) $\left(\frac{3}{5}\right)^{12}$ b) $\left(\frac{3}{5}\right)^{-12}$
 c) $\left(\frac{3}{5}\right)^{12}$ d) $\left(-\frac{3}{5}\right)^{12}$
8. a) $\frac{a^2}{b^2}$ b) $\frac{n^6}{m^3}$
 c) $\frac{a^8}{c^8}$ d) $\frac{4b^2}{25c^2}$
 e) a^2b^2 f) n^6m^3
 g) $\frac{1}{c^{12}d^8}$ h) $\frac{x^3}{y^3}$

9. a) x ; product of powers law
 b) a^{-5} ; product of powers law
 c) b^3 ; product of powers law
 d) 1; product of powers law
 e) $\frac{1}{x^7}$; quotient of powers law
 f) s^{10} ; quotient of powers law
 g) $\frac{1}{b^5}$; quotient of powers law
 h) 1; quotient of powers law

10. a) 2.25 b) $\frac{9}{16}$
 c) 0.36 d) 1
 e) $\frac{5}{3}$ f) $-\frac{3}{8}$
 g) $\frac{1000}{343}$ h) $\frac{3}{10}$
11. a) x^3y^6 b) $\frac{a^4}{4b^4}$
 c) $\frac{1}{64m^6n^9}$ d) $\frac{16m^8n^{12}}{81}$
12. 10.6 cm
 13. 251 ft.²
14. a) $\frac{a^5}{b}$ b) $\frac{d^4}{c^2}$
15. a) -32 b) $-\frac{1}{8}$
 c) $-\frac{1}{32}$ d) $\frac{1}{1024}$

16. a) m^2 b) $\frac{1}{x^{\frac{5}{4}}}$
- c) $\frac{3b^{\frac{1}{2}}}{a^6}$ d) $-\frac{4c^2b^6}{a^3}$
17. a) $\frac{x^{\frac{5}{2}}}{y^4}$ b) $\frac{b}{25a^4}$
19. a) $\frac{m^8}{n^2}$ b) $\frac{r^{\frac{1}{2}}}{s^4}$

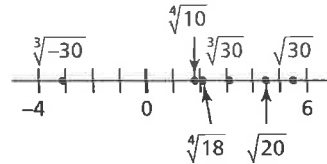
20. a) i) Dimensions, in millimetres: $\frac{1000}{2^4}$ by $\frac{1000}{2^4}$;
297 mm by 420 mm
- ii) Dimensions, in millimetres: $\frac{1000}{2^4}$ by $\frac{1000}{2^4}$;
210 mm by 297 mm
- iii) Dimensions, in millimetres: $\frac{1000}{2^4}$ by $\frac{1000}{2^4}$;
149 mm by 210 mm
- b) i) Dimensions, in millimetres: $\frac{1000}{2^4}$ by $\frac{1000}{2^4}$
- ii) Dimensions, in millimetres: $\frac{1000}{2^4}$ by $\frac{1000}{2^4}$
- iii) Dimensions, in millimetres: $\frac{1000}{2^4}$ by $\frac{1000}{2^4}$
- c) A piece of A4 paper has the same dimensions as a folded piece of A3 paper; a piece of A5 paper has the same dimensions as a folded piece of A4 paper.

21. a) $\frac{a^{16}c^3}{b^7}$ b) $\frac{c^{14}}{64a^3b^{10}}$
22. a) $\frac{1}{a^{\frac{10}{9}}}$ b) $\frac{1}{a^2}$

23. For example:
- a) $x^1 \cdot x^{\frac{1}{2}}$, $x^{\frac{3}{4}} \cdot x^{\frac{3}{4}}$, $x^2 \cdot x^{-\frac{1}{2}}$
- b) $x^2 \div x^{\frac{1}{2}}$, $x^{\frac{5}{2}} \div x^1$, $x^{-1} \div x^{-\frac{5}{2}}$
- c) $\left(x^{\frac{1}{2}}\right)^3$, $\left(x^6\right)^{\frac{1}{4}}$, $\left(x^{-\frac{1}{3}}\right)^{-\frac{9}{2}}$
24. $\frac{1}{2}\left(\frac{3}{2}\right)^{\frac{1}{2}}$ cm, or approximately 0.6 cm

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1. a) 10 b) 0.9
- c) 2 d) $\frac{3}{5}$
2. The index tells which root to take.
3. a) 3.3 b) -2.3
- c) 2.0
4. a) 25 b) 216
- c) 2401
5. Neither
6. a) Rational b) Rational
- c) Rational d) Irrational
- e) Rational f) Rational
- g) Rational h) Irrational
- i) Irrational
7. Approximately 4.8 cm
8. a) Rational b) Irrational
9. $\sqrt[3]{-30}$, $\sqrt[4]{10}$, $\sqrt[4]{18}$, $\sqrt[3]{30}$, $\sqrt{20}$, $\sqrt{30}$



10. 1 s
11. a) $5\sqrt{6}$ b) $3\sqrt[3]{5}$
- c) $4\sqrt{7}$ d) $3\sqrt[4]{2}$
12. a) $\sqrt{180}$ b) $\sqrt{126}$
- c) $\sqrt[3]{192}$ d) $\sqrt[4]{32}$
13. Approximately 1.0 cm
15. $6\sqrt{2}$, $3\sqrt{6}$, $5\sqrt{2}$, $4\sqrt{3}$, $2\sqrt{7}$
17. a) $\sqrt[4]{12}$ b) $\sqrt[3]{(-50)^5}$, or $(\sqrt[3]{-50})^5$
- c) $\sqrt{1.2}$ d) $\sqrt[3]{\frac{3}{8}}$
18. a) $1.4^{\frac{1}{2}}$ b) $13^{\frac{2}{3}}$
- c) $2.5^{\frac{4}{5}}$ d) $\left(\frac{2}{5}\right)^{\frac{3}{4}}$
19. a) 2 b) 1.2
- c) -32 d) $\frac{27}{64}$
20. Approximately 35%
21. $(\sqrt{5})^3$, $5^{\frac{3}{4}}$, $5^{\frac{2}{3}}$, $\sqrt[3]{5}$, $\sqrt[4]{5}$