Asymptotes: Horizontal and Otherwise

Goal:

- Can identify the vertical, horizontal, and slant asymptotes of a function
- Understand that asymptotes are the description of regular behaviour as something becomes infinite
- Gaining comfort to graph key characteristics of functions.

Terminology:

- Slant Asymptote
- Infinite Limit

Review: From grade 12 what is a horizontal and vertical asymptote?

Calculus: We want to ammend the above description to have a calculus perspective.

When we approach infinity with polynomials, we only care about relative growth. That is x^2 grows faster than x, and x^3 grows faster still.

$$\mathcal{O}(x^n) > \mathcal{O}(x^{n-1})$$

So for large values of x, only the leading term really maters

Example:

$$\lim_{x\to\infty} \frac{6x^4 + 6x^2 - 100}{2x^4 - 49x^3 + 10000}$$

Practice:

$$\lim_{x\to-\infty} \frac{4x^5 - 17x^3 + 400x - 20^{20}}{0.001x^6 - x^5 + 200x^2}$$

But what happens when the degree of the numerator is greater than the degree of the denominator?
Example:

$$\lim_{x\to\infty} \frac{2x^3 - x^2 + 400x}{x^2 + x + 1}$$

Practice:

$$\lim_{x\to\infty} \frac{7x^5 - 2x^4 - 10x^2 + 1}{x^4 + 1}$$

Practice Problems: 5.1: # Anything you feel is valuable (This section is Precalc 12 and early limit review)
5.2: # 1-3 (do what you need), 4, 6, 11
5.6: # 1-3


5.2 # 7-10

Desmos Asymptote Activity

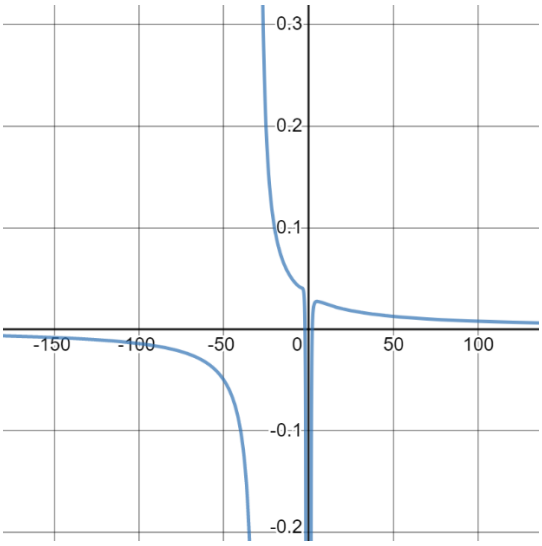
I want you to find the equation to the horizontal and slant asymptotes by using Desmos to graph and compare the rational function to the equation to the asymptote as I showed in class.

Go to:

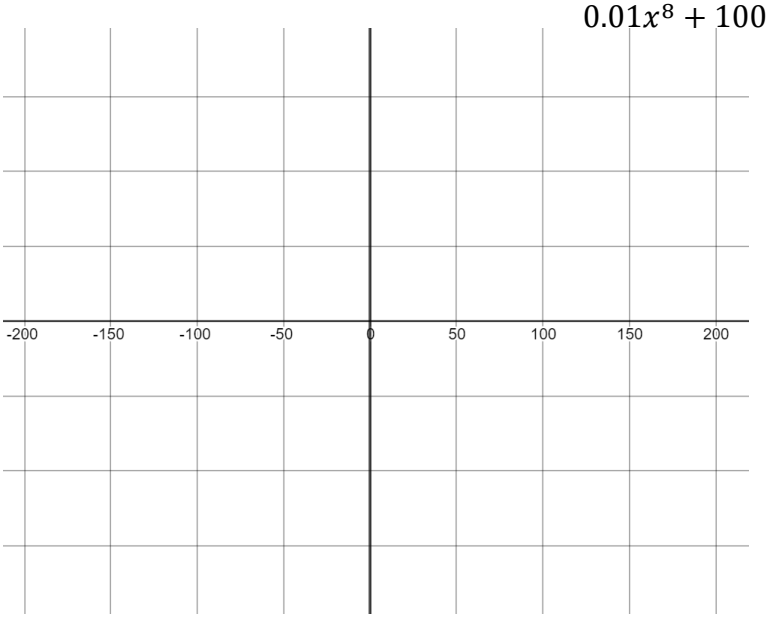
desmos.com/calculator/rhnw0r4upz

Find the equation to the horizontal/slant asymptote and graph the asymptote along with the graph.

1. $$\frac{x^6 - 5x^3 - 100}{x^7 + 30x^6}$$

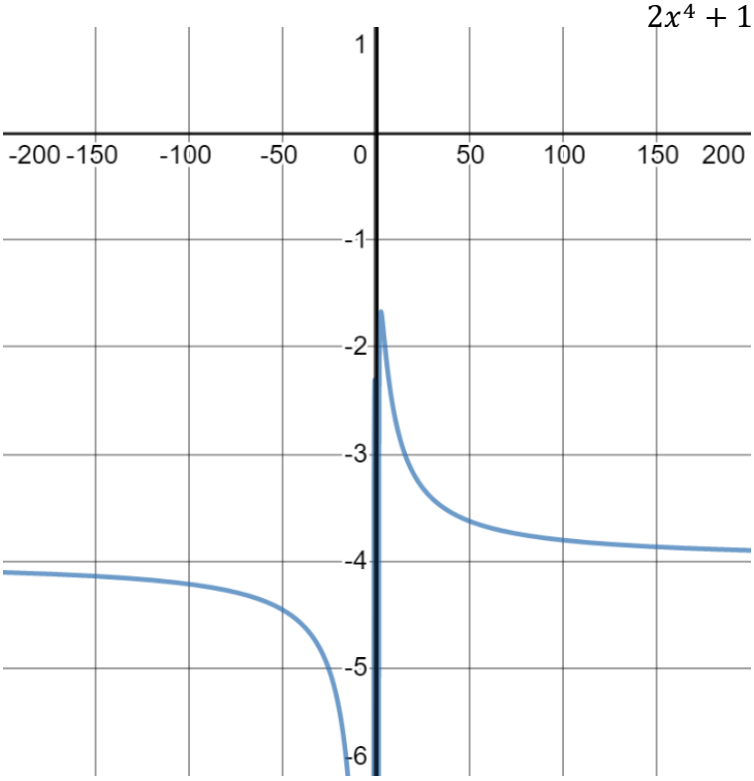


2. $$\frac{-x^5 + 100x^3 - 200}{0.01x^8 + 100}$$



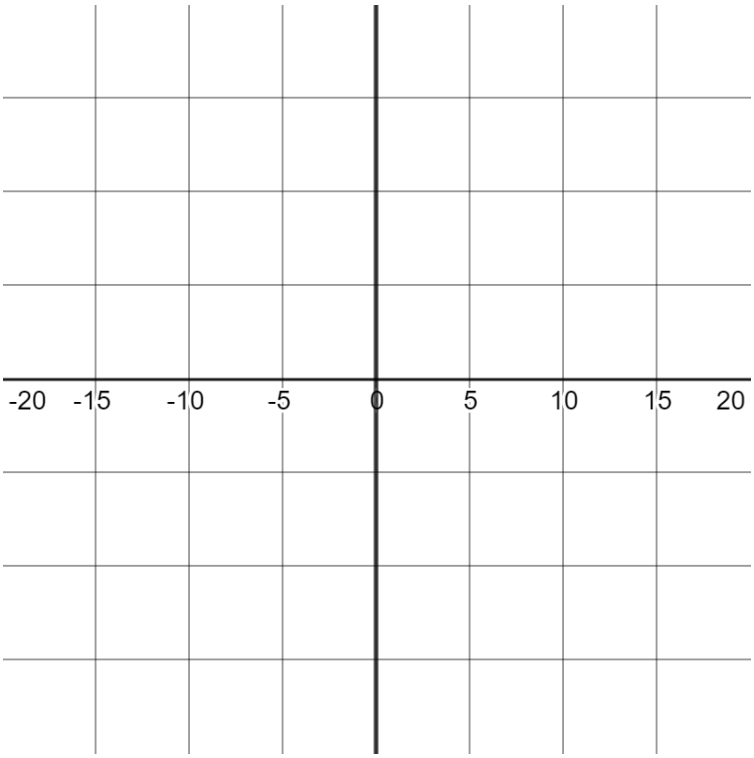
3.

$$\frac{-8x^4 + x^3 - 20}{2x^4 + 10x^3 - 18x}$$



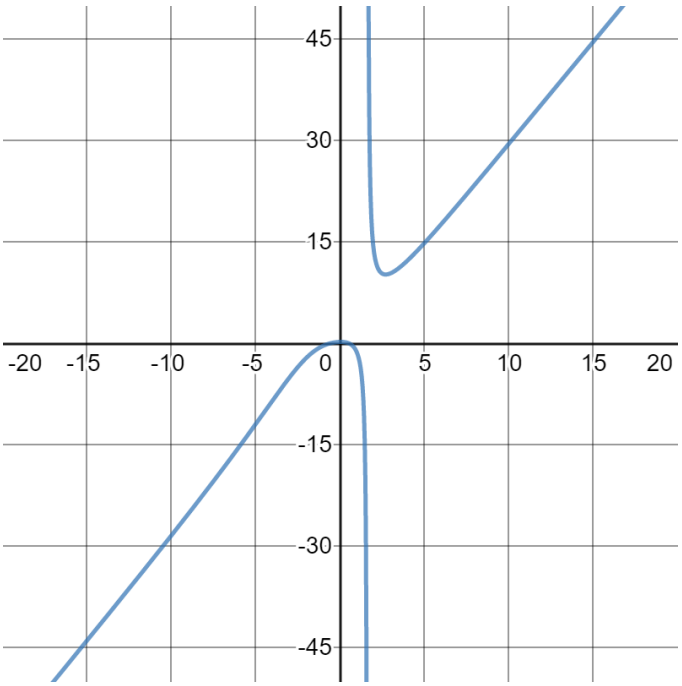
4.

$$\frac{20x^3 - 6x^2 + 15x}{4x^3 - 10x^2 + 1}$$



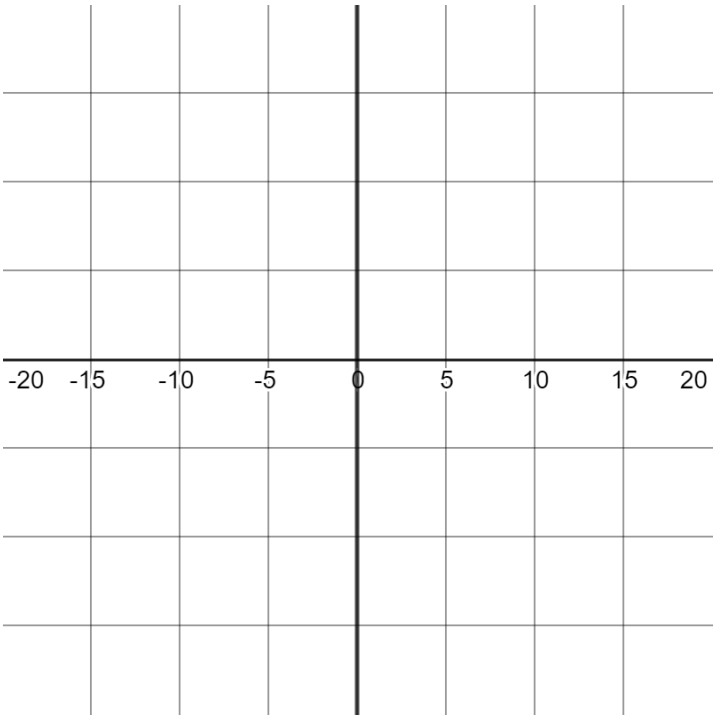
5.

$$\frac{3x^4 + 10x^2 - 3}{x^3 + 7x - 16}$$



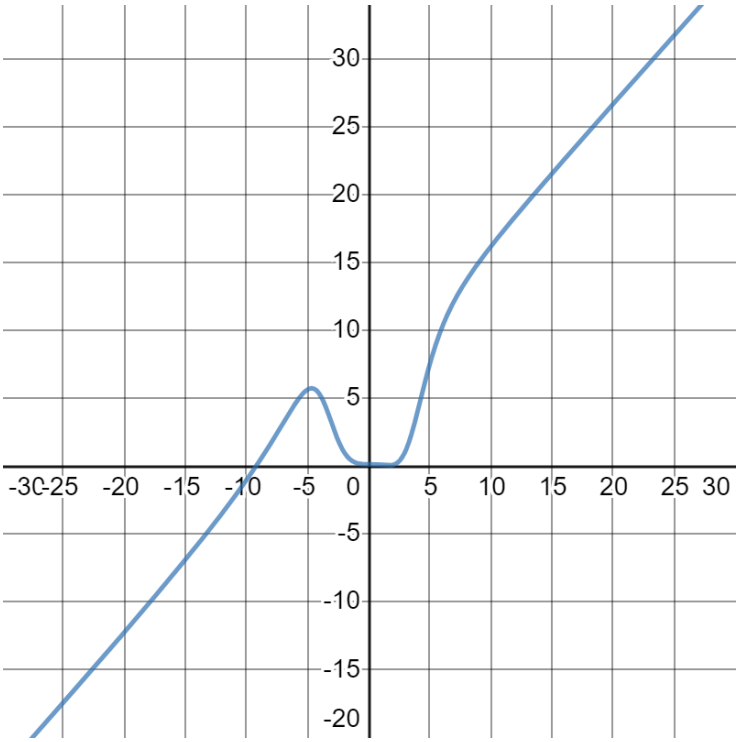
6.

$$\frac{-x^5 - 4x^3 + 20x}{2x^4 + 5x^2 - 10}$$



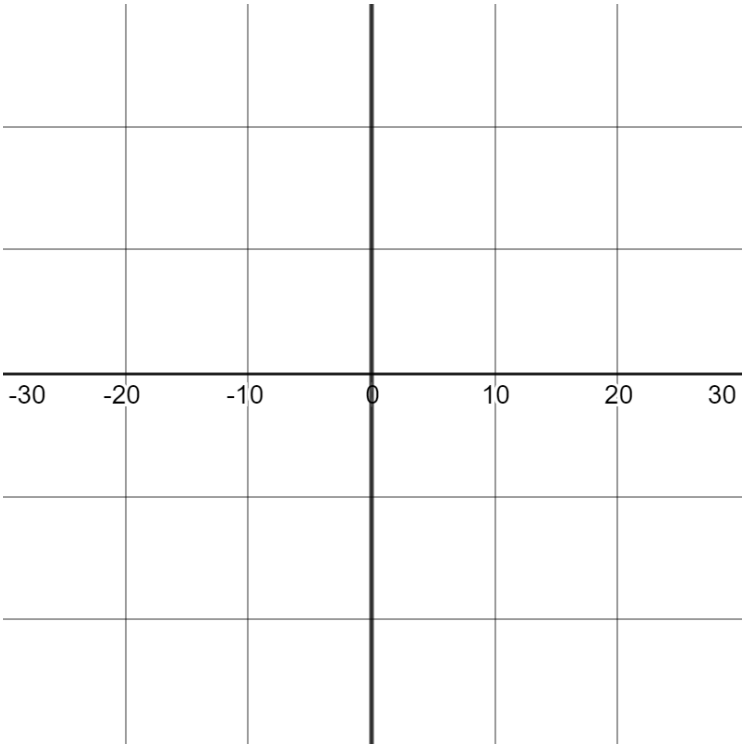
7.

$$\frac{x^5 + 7x^4 - 20x^3 + 30}{x^4 - 10x^2 + 300}$$



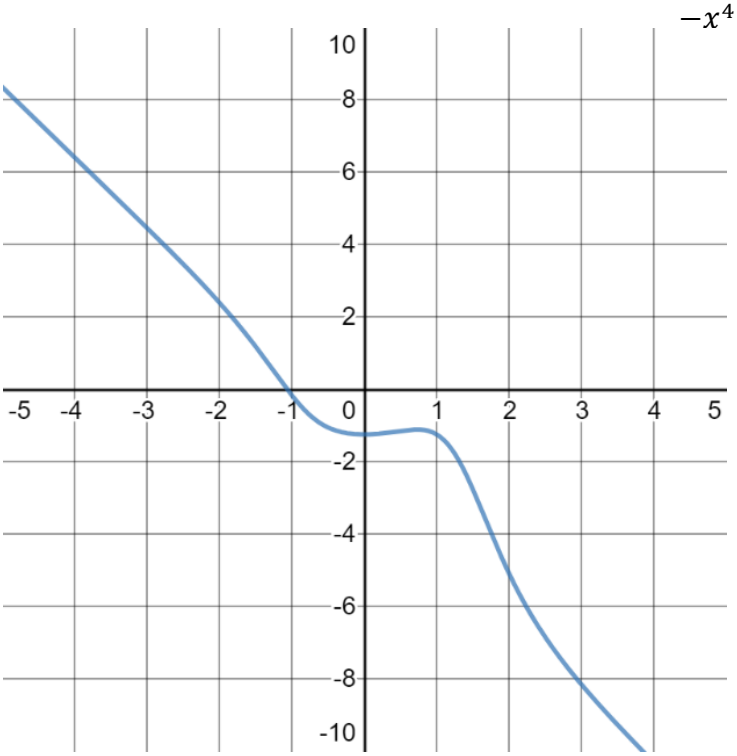
8.

$$\frac{2x^4 - 10x^3 + 30x^2 - 100}{x^3 + 29x}$$



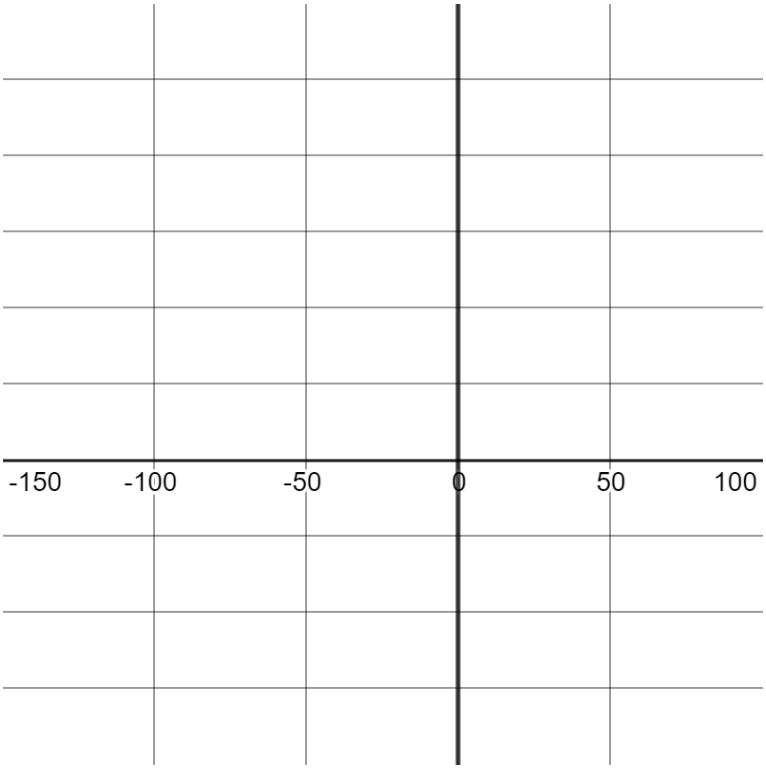
9.

$$\frac{2x^5 - 3x^2 + 5}{-x^4 + x^3 - 4}$$



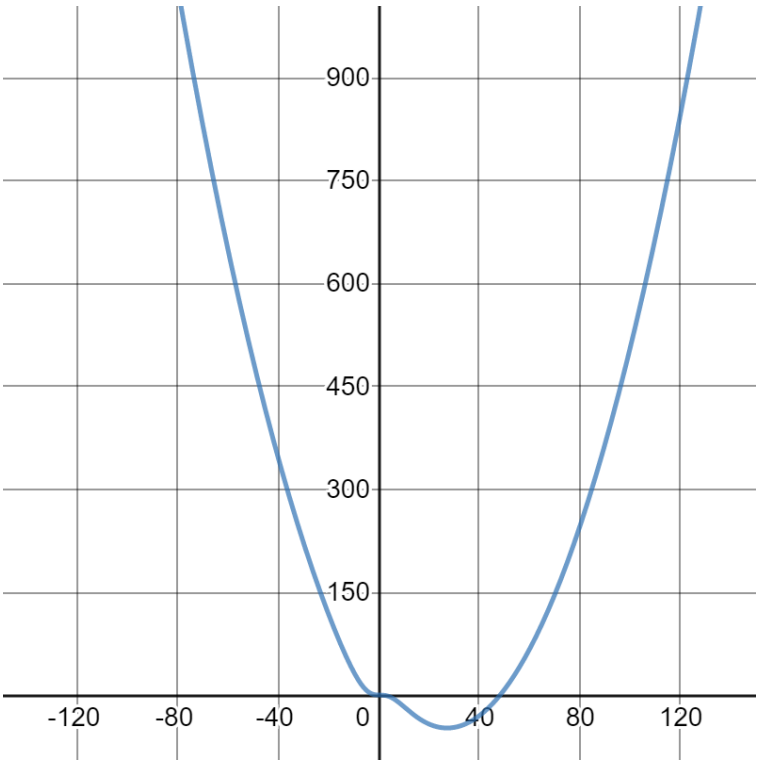
10.

$$\frac{-x^3 + 3x - 10}{3x^2 + 18x - 20}$$



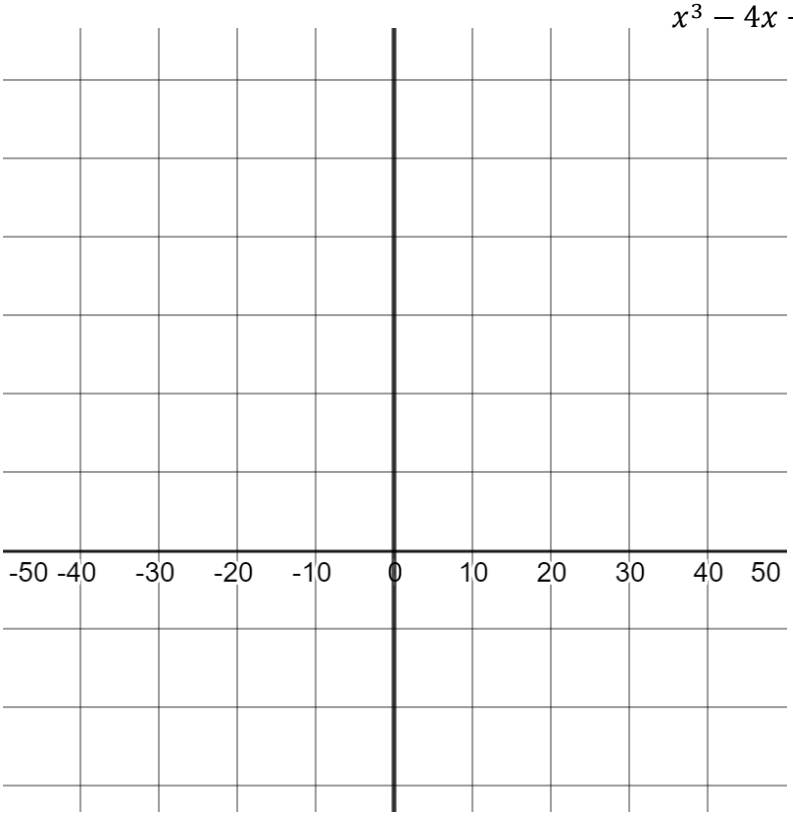
11.

$$\frac{0.1x^4 - 5x^3 + 8x^2 - 10x + 20}{x^2 + 100}$$



12.

$$\frac{(x^5 - 2x^4 + 5x^3 - 43x^2 + 30)}{x^3 - 4x + 100}$$



In general, what strategies are you using and what patterns are you noticing? Be as specific as possible.