Asymptotes: Horizontal and Otherwise

Goal:
- Can identify the vertical, horizontal, and slant asymptotes of a function
- Understand that asymptotes are the description of regular behaviour as something becomes infinite
- Gaining comfort to graph key characteristics of functions.

Terminology:
- Slant Asymptote
- Infinite Limit

Review: From grade 12 what is a horizontal and vertical asymptote?

Calculus: We want to amend the above description to have a calculus perspective.
When we approach infinity with polynomials, we only care about relative growth. That is $x^2$ grows faster than $x$, and $x^3$ grows faster still.

$$O(x^n) > O(x^{n-1})$$

So for large values of $x$, only the leading term really matters.

**Example:**

$$\lim_{{x \to \infty}} \frac{6x^4 + 6x^2 - 100}{2x^4 - 49x^3 + 10000}$$

**Practice:**

$$\lim_{{x \to -\infty}} \frac{4x^5 - 17x^3 + 400x - 20^{20}}{0.001x^6 - x^5 + 200x^2}$$
But what happens when the degree of the numerator is greater than the degree of the denominator?

**Example:**

\[
\lim_{x \to \infty} \frac{2x^3 - x^2 + 400x}{x^2 + x + 1}
\]

**Practice:**

\[
\lim_{x \to \infty} \frac{7x^5 - 2x^4 - 10x^2 + 1}{x^4 + 1}
\]

**Practice Problems:**

5.1: # Anything you feel is valuable (This section is Precalc 12 and early limit review)
5.2: # 1-3 (do what you need), 4, 6, 11
5.6: # 1-3

5.2 # 7-10
Desmos Asymptote Activity

I want you to find the equation to the horizontal and slant asymptotes by using Desmos to graph and compare the rational function to the equation to the asymptote as I showed in class.

Go to:

desmos.com/calculator/rhnw0r4upz

Find the equation to the horizontal/slant asymptote and graph the asymptote along with the graph.

1. \[
\frac{x^6 - 5x^3 - 100}{x^7 + 30x^6}
\]

2. \[
\frac{-x^5 + 100x^3 - 200}{0.01x^8 + 100}
\]
3. \(\frac{-8x^4 + x^3 - 20}{2x^4 + 10x^3 - 18x}\)

4. \(\frac{20x^3 - 6x^2 + 15x}{4x^3 - 10x^2 + 1}\)
5. \[\frac{3x^4 + 10x^2 - 3}{x^3 + 7x - 16}\]

6. \[\frac{-x^5 - 4x^3 + 20x}{2x^4 + 5x^2 - 10}\]
7. \[ \frac{x^5 + 7x^4 - 20x^3 + 30}{x^4 - 10x^2 + 300} \]

8. \[ \frac{2x^4 - 10x^3 + 30x^2 - 100}{x^3 + 29x} \]
9. \[ \frac{2x^5 - 3x^2 + 5}{-x^4 + x^3 - 4} \]

10. \[ \frac{-x^3 + 3x - 10}{3x^2 + 18x - 20} \]
11. \[
\frac{0.1x^4 - 5x^3 + 8x^2 - 10x + 20}{x^2 + 100}
\]
In general, what strategies are you using and what patterns are you noticing? Be as specific as possible.