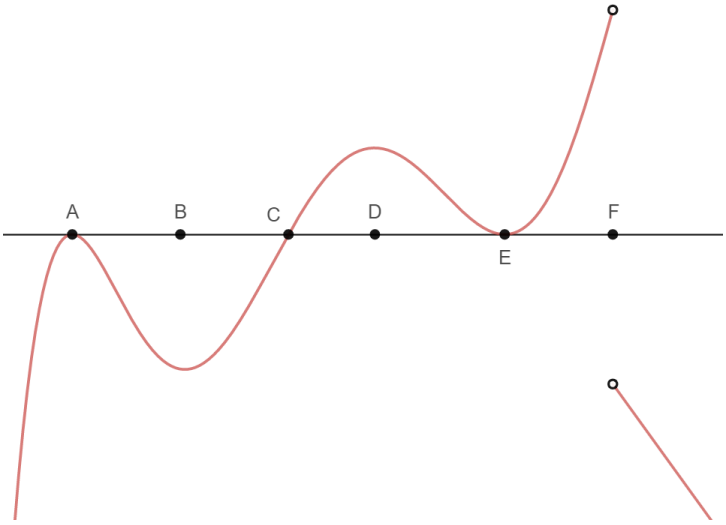
Second Derivative Test and Concavity

Goal: • Can determine the intervals of concavity of a function f by analyzing the function algebraically, or by analyzing the values of f'' given a graph or table. • Can determine the type of an extremum at a point using the second derivative test and concavity.
Terminology: • Concavity (concave up and down) • Inflection Point • Second Derivative Test
Reminder: • Quiz on Tuesday on Concavity and Slant/Horizontal Asymptotes

Review: Given the following graph of $\frac{df}{dx}$, determine the extrema of f and if they are a max or min.



Now that we know what the first derivative tells us (namely the slope and if the curve is going up or down). What does the second derivative tell us?

What if $f'' > 0$?

What if $f'' < 0$?

When f'' changes sign we have an **inflection point**: where we change from concave up to concave down.

NOTE: This means at $x = c,\ f''(c) = 0$ or could be undefined but f'' must go from positive to negative.

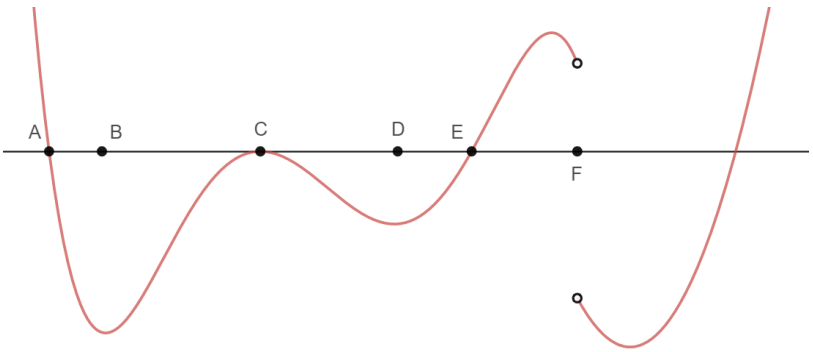
Example: Find the inflection points and intervals of concavity of the function:

$$f(x) = x^3 - 3x^2 + 9$$

Practice: Find the inflection points and intervals of concavity of the function:

$$g(x) = x^4 - 24x^2 + 10x$$

Practice: Given the graph of $\frac{d^2h}{dx^2}$, where are the inflection points and where is h concave up? Assume h is continuous.



We note that when $f'' > 0$ we could have

And if $f'' < 0$ then we could have

This builds the **Second Derivative Test!** If we know that $f'(c) = 0$ then we could have a

- If $f''(c) > 0$ then we have
- If $f''(c) < 0$ then we have
- If $f''(c) = 0$ then we have

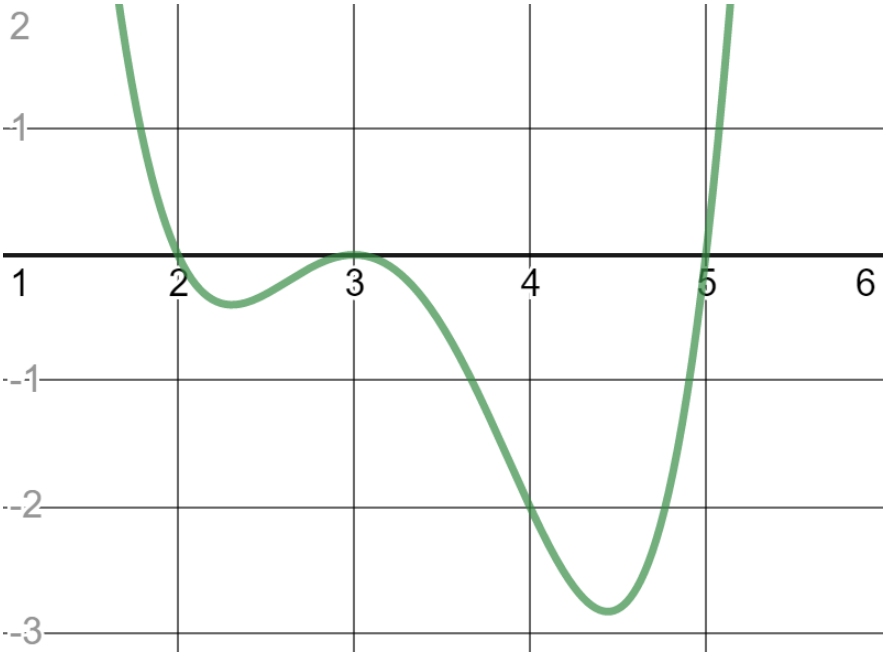
Example: Find the extrema of the following function using second derivative test.

$$k(x) = x^4 + 4x^3 - 7$$

Practice: Find the extrema of the following function using second derivative test.

x	$x < A$	A	$A < x < B$	B	$B < x < C$	C	$x > C$
$m''(x)$	Positive	0	Negative	undefined	Positive	0	Positive
$m'(x)$	0 For some $a < A$	5	0 For some $a_b \in (A, B)$	undefined	0 For some $b_c \in (B, C)$	0	0 For some $c > C$

Practice: Consider the graph of n'' below. If $n'(x) = 0$ when $x = 0.7, 4$, and 5.3 then determine where the extrema of n occur and the type of extrema.



Practice Problems: 5.3: # 1, 2a-f, 3-9
5.4: # 1 (what you need), 3abc