## **Sequences and Series**

KNOW	DO		UNDERSTAND
Sequences are functioning whose	Can use sigma notation to describe series. Can		None yet
domain is discrete. Can identify if a	build the equation for a geometric sequence.		
sequence is geometric.	Can determine the sum of a geometric series.		
Vocab & Notation			
• Sequence, $(a_n)_{n \in \mathbb{N}}$		Series	
• Index		• Sigma Notation, $\sum k$	
Recursion		Geometric	
		Common Ratio	

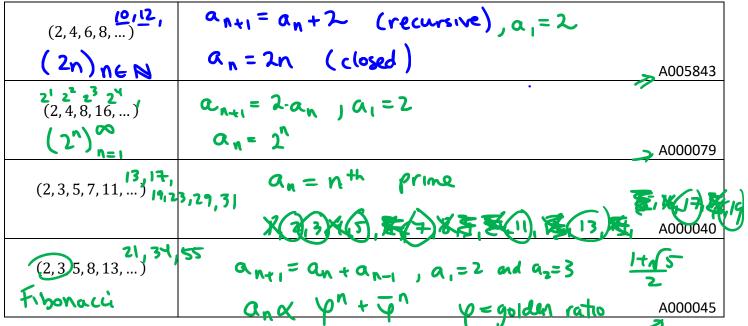
Definition: A sequence is an ordered collection of objects. Like a set, but order matters. It is denoted as

$$(a_n)_{n\in\mathbb{N}} = (a_1)a_2(a_3)...)$$

The position of an element of a sequence is called the **index**, and there is a relationship between the index and the **term**.

$$s: \mathbb{N} \to \mathbb{R}$$
$$s: n \mapsto a_n$$

Examples:



There is an online encyclopedia of integer sequences called OEIS and found at "oeis.org". The sequence number is shown on the side.

Oftentimes it is helpful to define a sequence starting with index 0 instead of 1. Really any number can be used to start the index but 0 and 1 are the main choices.

$$(2n)_{n=1}^{\infty} = (2m)_{m=1}^{\infty} = (2(k_{1}))_{k=0}^{\infty} = (2, 4, 6, ...)$$
  
let  $k = m - 1 \implies m = k + 1$   
if  $m = 1 \implies k = 0$ 

Definition: A geometric sequence is a sequence generated by multiplying the previous term by a fixed value.

$$a_{k+1} = a_k \cdot r$$

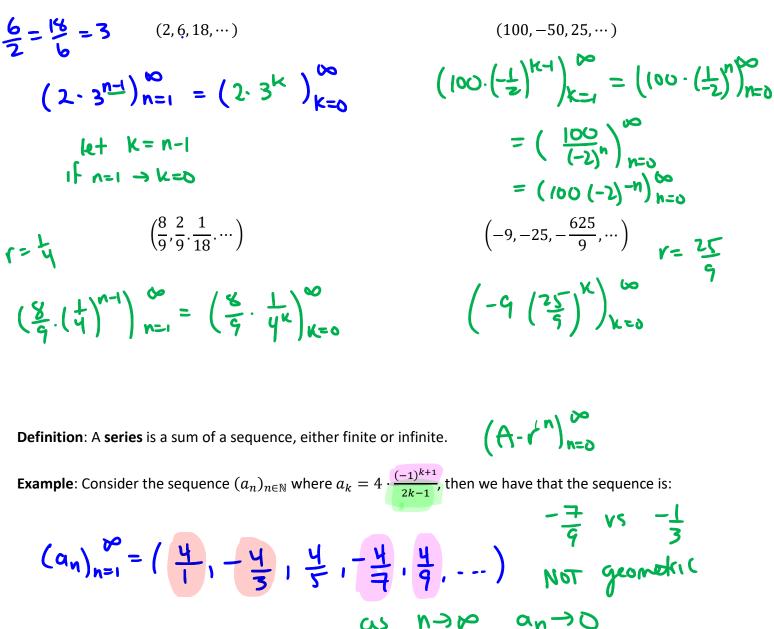
Definition: The common ratio is the ratio of consecutive terms in a geometric sequence.

true for the 
$$\frac{a_{k+1}}{a_k} = r$$

Let's look and see how a geometric sequence is built by letting  $a_1 = A$ 

$$(a_n)_{n\in\mathbb{N}} = (A, Ar, Ar^2, Ar^3, \dots, Ar^{k-1}, \dots) = (Ar^{n-1})_{n\in\mathbb{N}}$$
  
=  $(a_1, a_2, a_3, a_4, \dots, a_{k}, \dots)$ 

**Example**: Determine sequences of the form  $(a_k)_{k=1}^{\infty}$  and  $(b_k)_{k=0}^{\infty}$  for the following patterns:



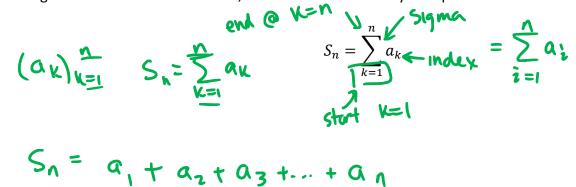
$$|+|+|+|+... = 00$$

Unit 4: Exponential Growth

And we can add the first 5 terms or add all of them as

$$S_5 = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} = \frac{1052}{315} = 3.3396 \dots$$
$$S_{\infty} = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \dots =$$

We'll get to the last sum in a second, but there is a better way to express the summation using sigma notation



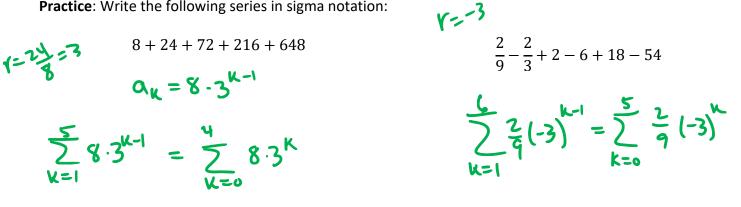
So the sums we were looking at can be written as

$$S_5 = \sum_{k=1}^{5} \frac{4 \cdot (-1)^{k+1}}{2k-1} \qquad \qquad S = \sum_{k=1}^{\infty} \frac{4 \cdot (-1)^{k+1}}{2k-1} \quad = \mathbf{1}$$

Typing this into Desmos we get the sigma notation by typing "sum".

Practice: Write out the terms of the following sums and compute the sum

Practice: Write the following series in sigma notation:



Now we would like to consider the geometric series, that is

$$S_n = \sum_{k=1}^n a_1 \cdot r^{k-1}$$

Oftentimes it is helpful to try and add or subtract copies of the summation to reduce it to something simpler.

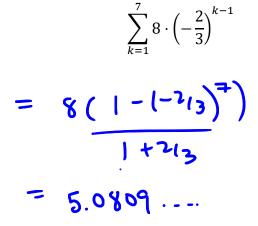
$$S_{n} = a_{1} + a_{1}r + a_{1}r^{2} + a_{1}r^{3} + \dots + a_{n}r^{n-1}$$

$$rS_{n} = a_{1}r + a_{1}r^{2} + a_{1}r^{3} + a_{1}r^{4} + \dots + a_{n}r^{n-1} + a_{n}r^{n}$$

$$(1-r)S_{n} = a_{1} - a_{n}r^{n} \implies S_{n} = a_{1}(1-r^{n})$$

$$f_{n} = a_{1}(1-r^{n})$$

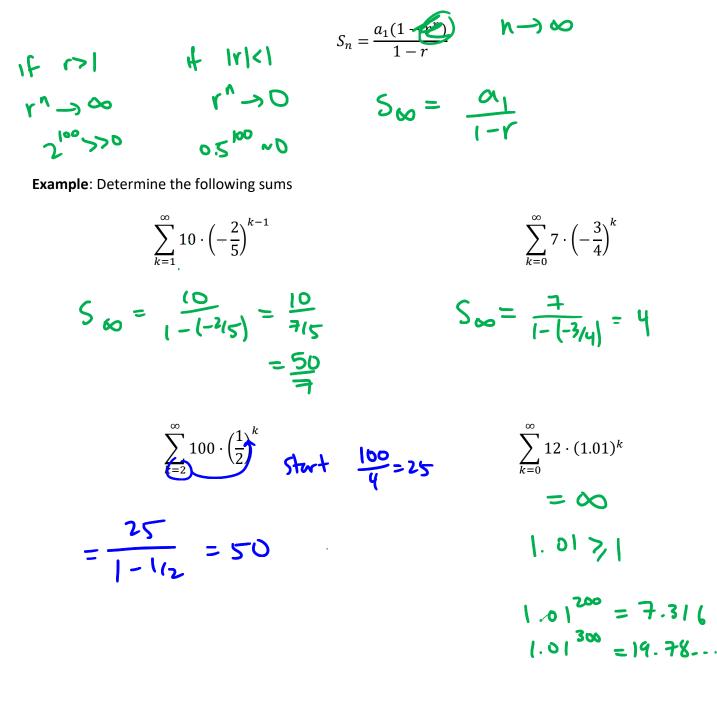
Practice: Determine the following sums



$$= \frac{1 - (9/4)^{2}}{(-9/4)^{2}}$$

$$= 232.743...$$

Since we know the finite sum, we can consider what  $S_{\infty}$  would be:



Practice Problems: Handout sections I, II, III, V(a,c,d)		
Sigma Notation Module:		
2 blogs.ubc.ca/infiniteseriesmodule/units/unit-1/sigma-notation/		
Ryerson pdf: 1.3 # 1-7, 23		
1.4 # 1-8, 16-19		
1.5 # 1-7, 20		