Unit 1 Progress Check: FRQ Part B

1. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

$$N(t) = \begin{cases} f(t) & \text{for } 0 \le t < 6 \\ 25t + 150 & \text{for } 6 \le t < 8 \\ \frac{200+80t}{2+0.05t} & \text{for } t \ge 8 \end{cases}$$

The number of fish in a pond at time t years is modeled by the function N defined above, where f is a continuous function such that $f(0) = 80$.

(a) Find $\lim_{t \to \infty} N(t)$. Explain the meaning of $\lim_{t \to \infty} N(t)$ in the context of the problem.

(b) Is the function N continuous at $t = 8$? Justify your answer.

(c) The function N is continuous at $t = 6$. Is there a time t, for $0 \le t \le 6$, at which $N(t) = 250$? Justify your answer.

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Please respond on separate paper, following directions from your teacher.

Part A

The second point requires a reference to time increasing and the connection to the number of fish.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✔

The student response accurately includes both of the criteria below.

- ☐ $\lim_{t \to \infty} N(t)$
- ☐ interpretation

Solution:

$\lim_{t \to \infty} N(t) = \lim_{t \to \infty} \frac{200+80t}{2+0.05t} = \frac{80}{0.05} = 1600$

As time increases, the number of fish in the pond approaches 1600.

Part B

At most 2 out of 3 points if mathematical notation for limits is missing or incorrect.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2	3 ✔

The student response accurately includes all three of the criteria below.

- ☐ $\lim_{t \to 8^-} N(t)$
- ☐ $\lim_{t \to 8^+} N(t)$
- ☐ answer with justification

Solution:

$\lim_{t \to 8^-} N(t) = \lim_{t \to 8^-} (25t + 150) = 350$

$\lim_{t \to 8^+} N(t) = \lim_{t \to 8^+} \frac{200+80t}{2+0.05t} = \frac{840}{2.4} = 350$

$N(8) = 350$

Yes, N is continuous at $t = 8$ because

$\lim_{t \to 8} N(t) = N(8)$.

Part C

At most 1 out of 2 points if correct justification does not specifically reference IVT

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2 ✔

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The student response accurately includes all of the criteria below.

- ☐ answer with justification
- ☐ specific reference to $\mathbf{IVT}$

Solution:

N is continuous for $0 \leq t \leq 6$.

$N\left(0\right) = 80 < 250 < 300 = N\left(6\right)$

By the Intermediate Value Theorem, there must be at least one time t, for $0 \leq t \leq 6$, such that $N\left(t\right) = 250$.

Part C

At most 1 out of 2 points if correct justification does not specifically reference $\mathbf{IVT}$

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	✔ 2

The student response accurately includes all of the criteria below.

- ☐ answer with justification
- ☐ specific reference to $\mathbf{IVT}$

Solution:

N is continuous for $0 \leq t \leq 6$.

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$N(0) = 80 < 250 < 300 = N(6)$

By the Intermediate Value Theorem, there must be at least one time t, for $0 \leq t \leq 6$, such that $N(t) = 250$.