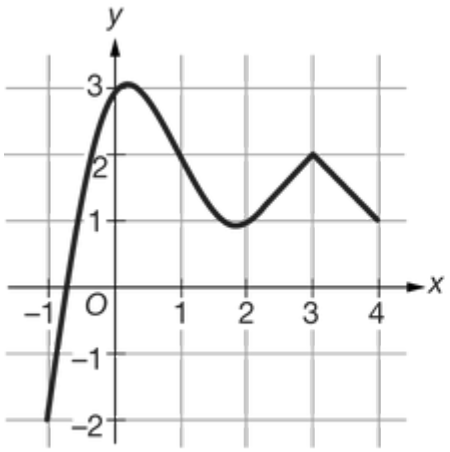
Unit 2 Progress Check: FRQ Part B

1. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

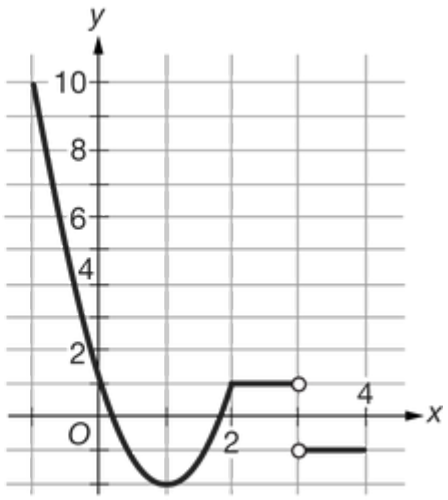
Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of f



Graph of f'

The graphs of the function f and its derivative f' are shown above for $-1 \le x \le 4$.

(a) Find the average rate of change of f over the interval $-1 \le x \le 4$. For how many values of x in the interval $-1 \le x \le 4$ does the instantaneous rate of change of f equal the average rate of change of f over that interval?

Please respond on separate paper, following directions from your teacher.

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Unit 2 Progress Check: FRQ Part B

(b) Write an equation for the line tangent to the graph of f at $x = 1$.

Please respond on separate paper, following directions from your teacher.

(c) For each of $\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$ and $\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$, find the value or give a reason why it does not exist.

Please respond on separate paper, following directions from your teacher.

(d) Let g be the function defined by $g(x) = e^x f(x)$. Find $g'(0)$.

Please respond on separate paper, following directions from your teacher.

Part A

If the average rate of change has a maximum of one sign error, the response is eligible for the second point based on a correct interpretation of the graph of f' with the presented average rate of change.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✔

The student response accurately includes both of the criteria below.

- ☐ average rate of change
- ☐ answer

Solution:

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Unit 2 Progress Check: FRQ Part B

$\frac{f(4)-f(-1)}{4-(-1)} = \frac{1-(-2)}{5} = \frac{3}{5}$

The average rate of change of f over the interval is $\frac{3}{5}$

$f'(x)$, the instantaneous rate of change of f at x, equals $\frac{3}{5}$ for two values of x in the interval $[-1, 4]$.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The student response accurately includes a correct tangent line equation.

Solution:

$f(1) = 2$

$f'(1) = -2$

An equation of the line tangent to the graph of f at $x = 1$ is $y = 2 - 2(x - 1)$.

Part C

The second point is earned for “does not exist” with a reason that indicates f is not differentiable at that point (e.g., indicates graph has a corner point or sharp point).

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

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Unit 2 Progress Check: FRQ Part B

The student response accurately includes both of the criteria below.

- ☐ $\lim_{x \to 2} \frac{f(x)-f(2)}{x-2}$
- ☐ $\lim_{x \to 3} \frac{f(x)-f(3)}{x-3}$ does not exist

Solution:

$\lim_{x \to 2} \frac{f(x)-f(2)}{x-2} = f'(2) = 1$

$\lim_{x \to 3} \frac{f(x)-f(3)}{x-3}$ does not exist because f is not differentiable at $x = 3.$

Part D

The first point is earned for a correct application of the product rule. The second point is earned for a correct expression that substitutes both correct numerical values.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✔

The student response accurately includes both of the criteria below.

- ☐ $g'(x)$
- ☐ answer

Solution:

$g'(x) = e^x f(x) + e^x f'(x)$

$g'(0) = e^0 f(0) + e^0 f'(0) = 3 + 1 = 4$

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Unit 2 Progress Check: FRQ Part B

Part D

The first point is earned for a correct application of the product rule. The second point is earned for a correct expression that substitutes both correct numerical values.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✔

The student response accurately includes both of the criteria below.

- ☐ $g'(x)$
- ☐ answer

Solution:

$g'(x) = e^x f(x) + e^x f'(x)$

$g'(0) = e^0 f(0) + e^0 f'(0) = 3 + 1 = 4$

2. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

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Unit 2 Progress Check: FRQ Part B

t (hours)	0	1	3	6	8
$P(t)$ (passengers)	0	35	204	600	728

The number of passengers who have boarded a ship is modeled by the differentiable function P, where t is the number of hours since boarding began. Values of $P(t)$ for selected values of t are given in the table above.

(a) According to the model, what is the average rate at which passengers board the ship, in passengers per hour, over the time interval $1 \leq t \leq 8$ hours?

Please respond on separate paper, following directions from your teacher.

(b) Write $P'(4.5)$ as the limit of a difference quotient. Use the data in the table to approximate $P'(4.5)$. Show the computations that lead to your answer.

Please respond on separate paper, following directions from your teacher.

(c) Must there be a time t, for $3 \leq t \leq 6$, at which $P(t) = 500$? Justify your answer.

Please respond on separate paper, following directions from your teacher.

(d) The total number of gallons of water used by the passengers on the ship is modeled by the function G defined by $G(t) = 120t\sqrt{t}$ for $0 \leq t \leq 8$, where t is the number of hours since boarding began. Find $G'(4)$, the rate at which passengers use water, in gallons per hour, at time $t = 4$ hours.

Please respond on separate paper, following directions from your teacher.

Part A

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Unit 2 Progress Check: FRQ Part B

The point is earned for a correct expression that substitutes both correct values from the table, with or without simplification.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✔

The student response accurately includes a correct average rate of change.

Solution:

$\frac{P(8)-P(1)}{8-1} = \frac{728-35}{7} = 99$

The average rate at which passengers board the ship over the time interval $1 \leq t \leq 8$ hours is 99 passengers per hour.

Part B

The second point is earned for a correct expression that substitutes both correct values from the table, with or without simplification.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✔

The student response accurately includes both of the criteria below.

- ☐ limit expression
- ☐ approximation

Solution:

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Unit 2 Progress Check: FRQ Part B

$P'(4.5) = \lim_{t \to 4.5} \frac{P(t) - P(4.5)}{t - 4.5}$

OR

$P'(4.5) = \lim_{h \to 0} \frac{P(4.5+h) - P(4.5)}{h}$

$P'(4.5) \approx \frac{P(6) - P(3)}{6 - 3} = \frac{600 - 204}{3} = 132$

Part C

1 out of 2 points if correct justification does not specifically reference IVT .

Select a point value to view scoring criteria, solutions, and/or examples and to score the response

0	1 ✔

The student response accurately includes one of the criteria below.

- ☐ P is continuous
- ☐ justification, using Intermediate Value Theorem

Solution:

Because P is differentiable, P is continuous.

$P(3) = 204 < 500 < 600 = P(6)$

By the Intermediate Value Theorem, there must be at least one time t, for $3 \leq t \leq 6$, such that

$P(t) = 500.$

Part D

The first point is earned for demonstration of a correct derivative for G and does not require a separate statement of $G'(t)$.

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Unit 2 Progress Check: FRQ Part B

The derivative and evaluation of the derivative can be expressed in one line to earn both points.

0	1	2 ✔

The student response accurately includes both of the criteria below.

- ☐ $G'(t)$
- ☐ $G'(4)$

Solution:

$G(t) = 120t\sqrt{t} = 120t^{\frac{3}{2}}$

$G'(t) = 120 \cdot \frac{3}{2}t^{\frac{1}{2}} = 180\sqrt{t}$

$G'(4) = 180\sqrt{4} = 180 \cdot 2 = 360$

At time $t = 4$ hours, passengers use water at a rate of 360 gallons per hour.

Part D

The first point is earned for demonstration of a correct derivative for G and does not require a separate statement of $G'(t)$.

The derivative and evaluation of the derivative can be expressed in one line to earn both points.

0	1	2 ✔

The student response accurately includes both of the criteria below.

- ☐ $G'(t)$

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Unit 2 Progress Check: FRQ Part B

☐ $G'(4)$

Solution:

$G(t) = 120t\sqrt{t} = 120t^{\frac{3}{2}}$

$G'(t) = 120 \cdot \frac{3}{2}t^{\frac{1}{2}} = 180\sqrt{t}$

$G'(4) = 180\sqrt{4} = 180 \cdot 2 = 360$

At time $t = 4$ hours, passengers use water at a rate of 360 gallons per hour.

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