Unit 3 Progress Check: FRQ Part A

1. A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.

You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. Your work must be expressed in standard mathematical notation rather than calculator syntax.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

x	-3	0	3	6
$f(x)$	-5	4	1	7
$f'(x)$	-1	2	-2	4

The table above gives values of a twice-differentiable function f and its first derivative f' for selected values of x. Let g be the function defined by $g(x) = f\left(x^2 - x\right)$.

(a) What is the value of $g'(3)$?

Please respond on separate paper, following directions from your teacher.

(b) It is known that $g''(0) = -1$. What is the value of $f''(0)$?

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Please respond on separate paper, following directions from your teacher.

(c) Is there a value c, for $0 < c < 3$, such that $g(c) = 5$? Justify your answer.

Please respond on separate paper, following directions from your teacher.

(d) Let h be the function with derivative given by $h'(x) = 4e^{\cos x}$. At what value of x in the interval $-3 \leq x \leq 0$ does the instantaneous rate of change of h equal the average rate of change of f over the interval $-3 \leq x \leq 0$?

Please respond on separate paper, following directions from your teacher.

Part A

Neither point may be earned for a response that does not apply the chain rule. The second point may be earned with a maximum of one error in $g'(x)$, unless the error is failing to apply the chain rule. Substitution of function values is required. A simplified answer is not required.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2 ✔

The student response accurately includes one of the criteria below.

- ☐ $g'(x)$
- ☐ $g'(3)$

Solution:

$g'(x) = f'\left(x^2 - x\right)(2x - 1)$

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$g'(3) = f'(6)(5) = 4(5) = 20$

Part B

Points may be earned in this part for consistent and correct use of the $g'(x)$ from part (a).

The first and second points require evidence of product rule and chain rule and no errors. At most 1 out of 3 points is earned for partial communication of product rule and chain rule with a maximum of one computational error. Substitution of function values is required. A simplified answer is not required.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The student response accurately includes all three of the criteria below.

- ☐ $g''(x)$ (2 points for full credit)
- ☐ answer

Solution:

$g''(x) = f''\left(x^2 - x\right)(2x-1)^2 + f'\left(x^2 - x\right)(2)$

$g''(0) = f''(0)(-1)^2 + f'(0)(2)$

$-1 = f''(0) + 2(2) \Rightarrow f''(0) = -5$

Part C

The second point may be earned if response contains conditions of IVT without naming the theorem.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

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The student response accurately includes both of the criteria below.

- ☐ g is continuous
- ☐ justification using Intermediate Value Theorem

Solution:

f is twice differentiable. $\Rightarrow f$ is continuous.

g is continuous as the composition of continuous functions.

$g(0) = f(0) = 4$ and

$g(3) = f(6) = 7.$

$\Rightarrow \ g(0) < 5 < g(3)$

By the Intermediate Value Theorem, there is a value of c, for $0 < c < 3$, such that $g(c) = 5.$

Part D

At most 1 out of 2 points is earned for an answer based on an average rate of change with a maximum of one computational error. Substitution of function values is required.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✔

The student response accurately includes both of the criteria below.

- ☐ average rate of change
- ☐ answer

Solution:

The average rate of change of f over the interval $-3 \leq x \leq 0$

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$\frac{f(0)-f(-3)}{0-(-3)} = \frac{4-(-5)}{3} = 3.$

$h'(x) = 4e^{\cos x} = 3$ for $-3 \le x \le 0. \Rightarrow x = -1.862602$

The instantaneous rate of change of h equals the average rate of change of f over the interval $-3 \le x \le 0$ at $x = -1.863$ (or -1.862).