Unit 3 Progress Check: FRQ Part B

1. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Consider the curve given by the equation $(2y+1)^3 - 24x = -3$.

(a) Show that $\frac{dy}{dx} = \frac{4}{(2y+1)^2}$.

Please respond on separate paper, following directions from your teacher.

(b) Write an equation for the line tangent to the curve at the point $(-1, -2)$.

Please respond on separate paper, following directions from your teacher.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point $(-1, -2)$.

Please respond on separate paper, following directions from your teacher.

(d) The point $\left(\frac{1}{6}, 0\right)$ is on the curve. Find the value of $\left(y^{-1}\right)'(0)$.

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Please respond on separate paper, following directions from your teacher.

Part A

The first point is earned for $3(2y+1)^2 \cdot 2\frac{dy}{dx} - 24 = 0$ or equivalent.

The second point cannot be earned without the first point. The second point is earned for a response that arrives at the given expression rather than an algebraically equivalent expression.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The student response accurately includes both of the criteria below.

- ☐ implicit differentiation
- ☐ verification

Solution:

$$\frac{d}{dx}\left((2y+1)^3 - 24x\right) = \frac{d}{dx}(-3)$$
$$\Rightarrow\ 3(2y+1)^2 \cdot 2\frac{dy}{dx} - 24 = 0$$
$$\Rightarrow\ \frac{dy}{dx} = \frac{24}{6(2y+1)^2} = \frac{4}{(2y+1)^2}$$

Part B

The second point may be earned if response contains an incorrect value for slope based on one computational error.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	✓ 2

The student response accurately includes both of the criteria below.

- ☐ slope
- ☐ equation for tangent line

Solution:

$\left.\frac{dy}{dx}\right|_{(x,\ y)=(-1,\ -2)} = \frac{4}{(2(-2)+1)^2} = \frac{4}{9}$

An equation for the tangent line is $y = -2 + \frac{4}{9}(x+1)$.

Part C

The first point is earned with $4(-2)(2y+1)^{-3} \cdot 2\frac{dy}{dx}$ or equivalent.

The second point is earned with an algebraic or numerical substitution for $\frac{dy}{dx}$. The response does not require an explicit expression for $\frac{d^2y}{dx^2}$. The third point does not require a simplified answer.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2	✓ 3

The student responses accurately includes all three of the criteria below.

- ☐ implicit differentiation (2 points for full credit)
- ☐ answer

Solution:

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$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(4(2y+1)^{-2}\right)$$
$$= 4(-2)(2y+1)^{-3} \cdot 2\frac{dy}{dx}$$
$$= \frac{-16}{(2y+1)^3} \cdot \frac{4}{(2y+1)^2} = \frac{-64}{(2y+1)^5}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(x,\ y)=(-1,\ -2)} = \frac{-64}{(2(-2)+1)^5} = \frac{-64}{-243} = \frac{64}{243}$$

Part D

The second point may be earned if response contains an incorrect value based on one computational error. Substitution of function values is required.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✔

The student response accurately includes both of the criteria below.

- ☐ $\frac{dy}{dx}$ at $\left(\frac{1}{6},\ 0\right)$
- ☐ answer

Solution:

$$\left.\frac{dy}{dx}\right|_{(x,\ y)=\left(\frac{1}{6},\ 0\right)} = \frac{4}{(2\cdot 0+1)^2} = 4$$

Because $\left(\frac{1}{6},\ 0\right)$ is on the curve, $\left(0,\ \frac{1}{6}\right)$ is on the inverse.

$$\left(y^{-1}\right)'(0) = \frac{1}{y'\left(y^{-1}(0)\right)} = \frac{1}{y'\left(\frac{1}{6}\right)} = \frac{1}{4}$$

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2. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

x	-3	-2	-1	1
$f(x)$	$-\frac{5}{2}$	-3	-2	$\frac{2}{3}$
$f'(x)$	-1	$\frac{1}{3}$	$\frac{6}{5}$	$\frac{4}{3}$

The table above gives values of the differentiable function f and its derivative for selected values of x.

(a) Let g be the function defined by $g(x) = \frac{f(x^2)}{e^x}$. Find $g'(-1)$.

Please respond on separate paper, following directions from your teacher.

Let h be the function defined by $h(x) = f(f(-2x))$. Find $h'(1)$.

Please respond on separate paper, following directions from your teacher.

Let k be the function defined by $k(x) = f(x) \cdot \arcsin\left(\frac{x}{2}\right)$. Find $k'(-1)$.

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Please respond on separate paper, following directions from your teacher.

Part A

The first and second points require evidence of quotient rule and chain rule and no errors. At most 1 out of 3 points is earned for partial communication of quotient rule and chain rule with a maximum of one computational error. Substitution of function values is required. A simplified answer is not required.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The student responses accurately includes all three of the criteria below.

- ☐ quotient rule
- ☐ chain rule
- ☐ answer

Solution:

$$g'(x) = \frac{f'(x^2) \cdot 2x \cdot e^x - f(x^2) \cdot e^x}{(e^x)^2}$$

$$g'(-1) = \frac{f'(1) \cdot 2(-1) \cdot e^{-1} - f(1) \cdot e^{-1}}{(e^{-1})^2}$$

$$= \frac{\frac{4}{3}(-2)e^{-1} - \frac{2}{3}e^{-1}}{(e^{-1})^2} = -\frac{10e}{3}$$

Part B

The first and second points require evidence of applying the chain rule twice and no errors. At most 1 out of 3 points is earned for partial communication of chain rule with a maximum of one computational error. Substitution of function values is required. A simplified answer is not required.

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✔

The student response accurately includes all three of the criteria below.

- ☐ $h'(x)$
- ☐ answer

Solution:

$h'(x) = f'(f(-2x)) \cdot f'(-2x) \cdot (-2)$

$h'(1) = f'(f(-2)) \cdot f'(-2) \cdot (-2)$
$= f'(-3) \cdot \frac{1}{3} \cdot (-2) = (-1) \cdot \frac{1}{3} \cdot (-2) = \frac{2}{3}$

Part C

The first and second points require evidence of product rule and chain rule and no errors. At most 1 out of 3 points is earned for partial communication of product rule and chain rule with a maximum of one computational error. Substitution of function values is required. A simplified answer is not required; trigonometric function values do not need to be evaluated.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2	3 ✔

The student response accurately includes all three of the criteria below.

- ☐ product rule
- ☐ chain rule

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☐ answer

Solution:

$$k'(x) = f'(x) \cdot \arcsin\left(\frac{x}{2}\right) + f(x) \cdot \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2}$$

$$k'(-1) = f'(-1) \cdot \arcsin\left(\frac{-1}{2}\right) + f(-1) \cdot \frac{1}{\sqrt{1-\left(\frac{-1}{2}\right)^2}} \cdot \frac{1}{2}$$

$$= \frac{6}{5} \cdot \left(-\frac{\pi}{6}\right) + (-2) \cdot \frac{2}{\sqrt{3}} \cdot \frac{1}{2} = -\frac{\pi}{5} - \frac{2}{\sqrt{3}}$$