

**Unit 4 Progress Check: FRQ Part B**

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**1. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.**

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

$$W(t) = \begin{cases} \frac{17}{2} - \frac{3}{2} \cos\left(\frac{\pi t}{6}\right) & \text{for } 0 \leq t \leq 6 \\ 10 - \frac{1}{5}(t-6)^2 & \text{for } 6 < t \leq 10 \end{cases}$$

The depth of a river at a certain point is modeled by the function  $W$  defined above, where  $W(t)$  is measured in feet and time  $t$  is measured in hours.

(a) Find  $W'(8)$ . Using correct units, explain the meaning of  $W'(8)$  in the context of the problem.



Please respond on separate paper, following directions from your teacher.

(b) The graph of  $W$  is concave down for  $3 \leq t \leq 3.5$ . Use the line tangent to the graph of  $W$  at  $t = 3$  to show that  $W(3.5) \leq 9$ .



Please respond on separate paper, following directions from your teacher.

(c) Find  $\lim_{t \rightarrow 2} \frac{W(t) - t^3 + \frac{1}{4}}{t-2}$ .



**Unit 4 Progress Check: FRQ Part B**

Please respond on separate paper, following directions from your teacher.

**Part A**

The first 2 points may be earned with the presence of a correct numerical answer.

The second point may be earned with a maximum of one sign error in  $W'(t)$ .

The third point requires a time reference, rate, and units. The response is eligible for the third point if the second point is earned AND provided the behavior described is consistent with the sign of the approximation, the numerical value, and the time reference.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1	2	3
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The student response accurately includes all three of the criteria below.

- $W'(t)$
- $W'(8)$
- interpretation with units

**Solution:**

For  $6 < t < 10$ ,  $W'(t) = -\frac{2}{5}(t - 6)$ .

$$W'(8) = -\frac{2}{5}(8 - 6) = -\frac{4}{5}$$

At time  $t = 8$  hours, the depth of the river is changing at a rate of  $-\frac{4}{5}$  foot per hour.

**Part B**

The second point may be earned with a maximum of one error in  $W'(t)$ .

For the second and third points, trigonometric function values do not need to be evaluated.



## Unit 4 Progress Check: FRQ Part B

The third point may be earned if the response is consistent with previous results.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 
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The student response accurately includes all four of the criteria below.

- $W'(t)$
- $W'(3)$
- approximation
- explanation

### Solution:

$$W(3) = \frac{17}{2} - \frac{3}{2} \cos\left(\frac{3\pi}{6}\right) = \frac{17}{2}$$

For  $0 < t < 6$ ,

$$W'(t) = \frac{3}{2} \sin\left(\frac{\pi t}{6}\right) \cdot \frac{\pi}{6} = \frac{\pi}{4} \sin\left(\frac{\pi t}{6}\right).$$

$$W'(3) = \frac{\pi}{4} \sin\left(\frac{3\pi}{6}\right) = \frac{\pi}{4}$$

$$W(3.5) \approx \frac{17}{2} + \frac{\pi}{4}(3.5 - 3) = \frac{17}{2} + \frac{\pi}{8}$$

Because the graph of  $W$  is concave down for  $3 \leq t \leq 3.5$ , approximation of  $W(3.5)$  is an overestimate of the actual value of  $W(3.5)$ .

$$\text{Therefore, } W(3.5) < \frac{17}{2} + \frac{\pi}{8} < \frac{17}{2} + \frac{1}{2} = 9.$$

### Part C

The first point requires showing that an indeterminate form  $\frac{0}{0}$  is present in order to apply L'Hospital's Rule. Any error in mathematical communication (e.g., writing " $= \frac{0}{0}$ " or not using limit notation) impacts



## Unit 4 Progress Check: FRQ Part B

the first point.

The second point requires substitution of function values. A simplified answer is not required; trigonometric function values do not need to be evaluated. Any differentiation or computation error impacts the second point.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2
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✓

The student response accurately includes both of the criteria below.

- applies L'Hospital's Rule
- answer

**Solution:**

$$\begin{aligned} \lim_{t \rightarrow 2} \left( W(t) - t^3 + \frac{1}{4} \right) &= W(2) - 2^3 + \frac{1}{4} \\ &= \frac{17}{2} - \frac{3}{2} \cos\left(\frac{2\pi}{6}\right) - 8 + \frac{1}{4} = 0 \end{aligned}$$

$$\lim_{t \rightarrow 2} (t - 2) = 0$$

By L'Hospital's Rule,

$$\begin{aligned} \lim_{t \rightarrow 2} \frac{W(t) - t^3 + \frac{1}{4}}{t - 2} &= \lim_{t \rightarrow 2} \frac{W'(t) - 3t^2}{1} = W'(2) - 3 \cdot 2^2 \\ &= \frac{\pi}{4} \sin\left(\frac{2\pi}{6}\right) - 12 = \frac{\pi\sqrt{3}}{8} - 12 \end{aligned}$$

**2. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.**

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the



**Unit 4 Progress Check: FRQ Part B**

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correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

A particle moves along the  $x$ -axis so that its position at time  $t > 0$  is given by  $x(t) = \frac{t^2 - 9}{3t^2 + 8}$ .

(a) Show that the velocity of the particle at time  $t$  is given by  $v(t) = \frac{70t}{(3t^2 + 8)^2}$ .



Please respond on separate paper, following directions from your teacher.

(b) Is the particle moving toward the origin or away from the origin at time  $t = 2$ ? Give a reason for your answer.



Please respond on separate paper, following directions from your teacher.

(c) The acceleration of the particle is given by  $a(t)$ . Write an expression for  $a(t)$ , and find the value of  $a(2)$ .



Please respond on separate paper, following directions from your teacher.

(d) What position does the particle approach as  $t$  approaches infinity?



Please respond on separate paper, following directions from your teacher.



## Unit 4 Progress Check: FRQ Part B

### Part A

The first point is earned for successful demonstration of the quotient rule.

The second point is earned for a response that arrives at the given expression rather than an algebraically equivalent expression.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

✓

0	1	2
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The student response accurately includes both of the criteria below.

- quotient rule
- verification

### Solution:

$$v(t) = x'(t) = \frac{2t(3t^2+8) - (t^2-9)6t}{(3t^2+8)^2}$$

$$= \frac{6t^3+16t-6t^3+54t}{(3t^2+8)^2} = \frac{70t}{(3t^2+8)^2}$$

### Part B

The response is eligible for the third point if either of the first 2 points is earned and the answer with reason is consistent with the previous results.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

✓

0	1	2	3
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The student response accurately includes all three of the criteria below.



**Unit 4 Progress Check: FRQ Part B**

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- $x(2) < 0$
- $v(2) > 0$
- answer with reason

**Solution:**

$$x(2) = \frac{-5}{20} = -\frac{1}{4} < 0$$

$$v(2) = \frac{140}{20^2} = \frac{7}{20} > 0$$

Because  $x(2) < 0$ , the particle is to the left of the origin at time  $t = 2$ .

Because  $v(2) > 0$ , the particle is moving to the right at time  $t = 2$ .

Therefore, the particle is moving toward the origin at time  $t = 2$ .

**Part C**

The first and second points require evidence of quotient rule and chain rule and no errors. At most 1 out of 3 points is earned for partial communication of quotient rule and chain rule with a maximum of one computational error.

A simplified answer is not required for the third point.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2	3 
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The student response accurately includes all three of the criteria below.

- quotient rule
- chain rule
- answer



**Unit 4 Progress Check: FRQ Part B**

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**Solution:**

$$\begin{aligned} a(t) = v'(t) &= \frac{70(3t^2+8)^2 - 70t \cdot 2(3t^2+8) \cdot 6t}{(3t^2+8)^4} \\ &= \frac{70(3t^2+8) - 70 \cdot 12t^2}{(3t^2+8)^3} = \frac{70(8-9t^2)}{(3t^2+8)^3} \end{aligned}$$

$$a(2) = \frac{70(8-9(2)^2)}{(3(2)^2+8)^3} = \frac{70(-28)}{20^3} = -\frac{49}{200}$$

**Part D**

Supporting work is not required to earn the point.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.



0	1
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The student response accurately includes a correct position.

**Solution:**

$$\lim_{t \rightarrow \infty} x(t) = \lim_{t \rightarrow \infty} \frac{t^2 - 9}{3t^2 + 8} = \frac{1}{3}$$

The particle approaches  $x = \frac{1}{3}$  as  $t$  approaches infinity.

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