Unit 5 Progress Check: MCQ Part A

1. Let f be the function given by $f(x) = 5\cos^2\left(\frac{x}{2}\right) + \ln(x+1) - 3$. The derivative of f is given by $f'(x) = -5\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right) + \frac{1}{x+1}$. What value of c satisfies the conclusion of the Mean Value Theorem applied to f on the interval $[1, 4]$?

(A) 2.132 because $f(2.132) = \frac{f(4)-f(1)}{3}$

(B) 2.749 because $f'(2.749) = \frac{f(4)-f(1)}{3}$ ✔

(C) 3.042 because $f'(3.042) = 0$

(D) 3.252 because $f'(3.252) = \frac{f(1)+f(4)}{2}$

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2. The derivative of the function f is given by $f'(x) = x^2 - 2 - 3x\cos x$. On which of the following intervals in $[-4, 3]$ is f decreasing?

(A) $[-4, -3.444]$, $[-1.806, -0.660]$, and $[1.509, 3]$

(B) $[-4, -2.805]$ and $[-1.227, 0.637]$

(C) $[-3.444, -1.806]$ and $[-0.660, 1.509]$ ✔

(D) $[-2.805, -1.227]$ and $[0.637, 3]$

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3. The temperature inside a vehicle is modeled by the function f, where $f(t)$ is measured in degrees Fahrenheit and t is measured in minutes. The first derivative of f is given by $f'(t) = t^2 - 3t + \cos t$. At what times t, for $0 < t < 4$, does the temperature attain a local minimum?

(A) 0.354 only

(B) 1.962

(C) 3.299 only ✔

(D) 0.354 and 3.299

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4. Let f be the function given by $f(x) = \frac{x}{(x-4)(x+2)}$ on the closed interval $[-7, 7]$. Of the following intervals, on which can the Mean Value Theorem be applied to f ?
1. $[-1, 3]$ because f is continuous on $[-1, 3]$ and differentiable on $(-1, 3)$.
2. $[5, 7]$ because f is continuous on $[5, 7]$ and differentiable on $(5, 7)$.
3. $[1, 5]$ because f is continuous on $[1, 5]$ and differentiable on $(1, 5)$.

(A) None

(B) I only

(C) I and II only ✔

(D) I, II, and III

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5. Let f be a differentiable function with $f(0) = -4$ and $f(10) = 11$. Which of the following must be true for some c in the interval $(0, 10)$?

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(A) $f'(c) = 0$, since the Extreme Value Theorem applies.

(B) $f'(c) = \frac{11+(-4)}{10-0}$, since the Mean Value Theorem applies.

(C) $f'(c) = \frac{11-(-4)}{10-0}$, since the Mean Value Theorem applies. ✔

(D) $f'(c) = 1.5$, since the Intermediate Value Theorem applies.

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6. Let f be the function given by $f(x) = \frac{x+4}{(x-1)(x+3)}$ on the closed interval $[-5, 5]$. On which of the following closed intervals is the function f guaranteed by the Extreme Value Theorem to have an absolute maximum and an absolute minimum?

(A) $[-5, 5]$

(B) $[-3, 1]$

(C) $[-2, 0]$ ✔

(D) $[0, 5]$

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7. Let f be the function defined by $f(x) = x \sin x$ with domain $[0, \infty)$. The function f has no absolute minimum and no absolute maximum on its domain. Why does this not contradict the Extreme Value Theorem?

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(A) The domain of f is not an open interval.

(B) The domain of f is not a closed and bounded interval. ✔

(C) The function f is not continuous on its domain.

(D) The function f is not differentiable on its domain.

8.

x	2	3	4	5
$f(x)$	1	14	20	31

Selected values of a continuous function f are given in the table above. Which of the following statements could be false?

(A) By the Intermediate Value Theorem applied to f on the interval $[2, 5]$, there is a value c such that $f(c) = 10$.

(B) By the Mean Value Theorem applied to f on the interval $[2, 5]$, there is a value c such that $f'(c) = 10$. ✔

(C) By the Extreme Value Theorem applied to f on the interval $[2, 5]$, there is a value c such that $f(c) \leq f(x)$ for all x in $[2, 5]$.

(D) By the Extreme Value Theorem applied to f on the interval $[2, 5]$, there is a value c such that $f(c) \geq f(x)$ for all x in $[2, 5]$.

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9. Let f be the function defined by $f(x) = x^3 - 6x^2 + 9x + 4$ for $0 < x < 3$. Which of the following statements is true?

(A) f is decreasing on the interval $(0, 1)$ because $f'(x) < 0$ on the interval $(0, 1)$.

(B) f is increasing on the interval $(0, 1)$ because $f'(x) < 0$ on the interval $(0, 1)$.

(C) f is decreasing on the interval $(0, 2)$ because $f''(x) < 0$ on the interval $(0, 2)$.

(D) f is decreasing on the interval $(1, 3)$ because $f'(x) < 0$ on the interval $(1, 3)$. ✔

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10. Let f be the function defined by $f(x) = x \ln x$ for $x > 0$. On what open interval is f decreasing?

(A) $0 < x < \frac{1}{e}$ only ✔

(B) $0 < x < 1$

(C) $x > \frac{1}{e}$

(D) There is no such interval.

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11. Let f be a function with first derivative given by $f'(x) = x(x-5)^2(x+1)$. At what values of x does f have a relative maximum?

(A) -1 only ✔

(B) 0 only

(C) -1 and 5 only

(D) -1, 0, and 5 only

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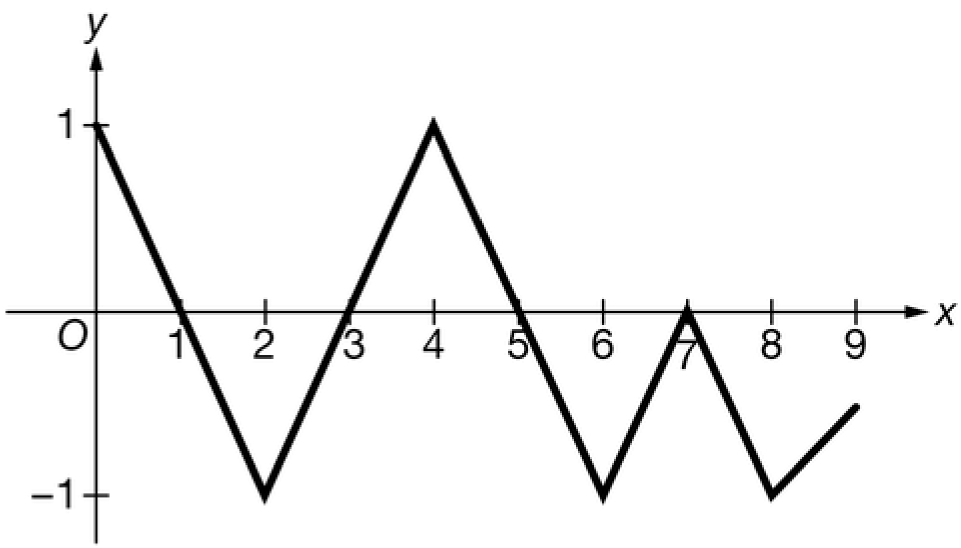
12.



Graph of f'

The graph of f', the derivative of the function f, is shown above for $0 < x < 9$. Which of the following statements is true for $0 < x < 9$?

(A) f has one relative minimum and two relative maxima. ✔

(B) f has two relative minima and one relative maximum.

(C) f has two relative minima and two relative maxima.

(D) f has three relative minima and two relative maxima.