Solving Sinusoidal Functions

KNOW	DO	UNDERSTAND
There are multiple solutions to a trig equation.	Can find the solutions to a trig equation in a given domain. Can use special triangles when appropriate.	*Inverse:* Sine and cosine are not 1-to-1 so the domain must be restricted. Restrictions come so that they take on all values of the range once.

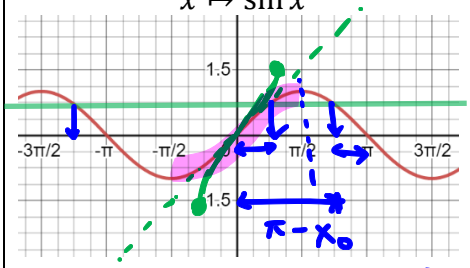
Vocab & Notation

- $\arcsin x$, $\arccos x$, $\arctan x$

Note how the domain gets restricted for the inverse functions:

Period is π

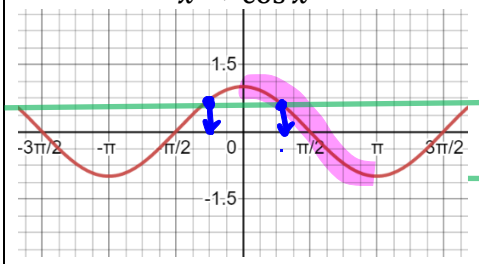
$f: \mathbb{R} \to [-1,1]$
$x \mapsto \sin x$



$f^{-1}: [-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$

* If x_0 is a solution so is $\pi - x_0$ $(x_0 > 0)$ or $-\pi - x_0$ $(x_0 < 0)$

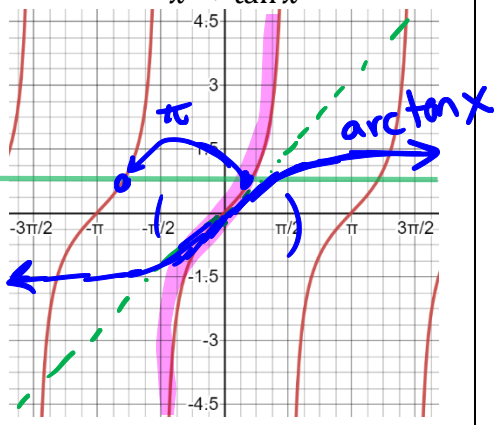
$g: \mathbb{R} \to [-1,1]$
$x \mapsto \cos x$



$g^{-1}: [-1,1] \to [0, \pi]$

* B/c cosine is even if x_0 is a solution so is $-x_0$

$h: \mathbb{R} \backslash \{x \mid \cos x = 0\} \to \mathbb{R}$
$x \mapsto \tan x$



$h^{-1}: \mathbb{R} \to (-\frac{\pi}{2}, \frac{\pi}{2})$

Example: $\arcsin(\sin x = 0.8)$

$x = \arcsin 0.8$
$= \sin^{-1}(0.8) = 0.927$
or $\pi - 0.927$

$\Rightarrow x = 0.927 + 2\pi n$
OR
$2.214 + 2\pi n$
$n \in \mathbb{Z}$

Example: $\arccos(\cos x = 0.8)$

$x = \arccos 0.8$
$= \pm 0.644$

$\Rightarrow x = \pm 0.644 + 2\pi n$

$n \in \mathbb{Z}$

Example: $\arctan(\tan x = 0.8)$

$x = \arctan 0.8$
$= 0.675$

$\Rightarrow x = 0.675 + \pi n$

$n \in \mathbb{Z}$

Example (With Calculator) Use algebra to solve the following trig equations:

$$\frac{1}{2}\sin(\pi(x-0.1)) = 0.2$$

θ

$T = \frac{2\pi}{\pi} = 2$

$\frac{1}{2}\sin\theta = 0.2$

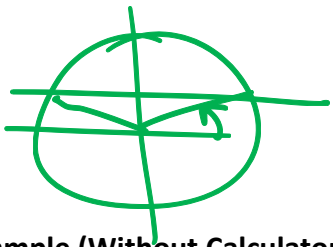
$\arcsin(\sin\theta = 0.4)$

$\Rightarrow \theta = 0.412$ or $\pi - 0.412 + 2\pi n$

$\Rightarrow \pi(x-0.1) = (0.412 \text{ or } 2.730) + 2\pi n$

period

$\Rightarrow x - 0.1 = (0.131 \text{ or } 0.869) + 2n$

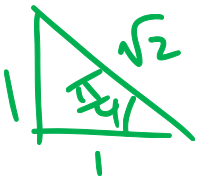


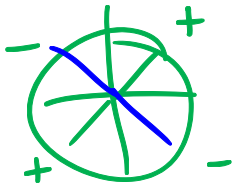
$x = 0.231 + 2n$ or $x = 0.969 + 2n$

Example (Without Calculator)

$$\left(\tan^2\left(\frac{1}{2}\left(x+\frac{\pi}{3}\right)\right)-1\right)\left(2\cos\left(\frac{x}{3}\right)+1\right)=0$$

θ φ





$(\tan^2\theta - 1)(2\cos\varphi + 1) = 0$

$=0$ $=0$

$\Rightarrow \tan^2\theta = 1$ or $\cos\varphi = -\frac{1}{2}$

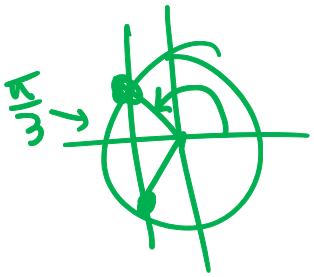
$\tan\theta = \pm 1$

$\Rightarrow \theta = \frac{\pi}{4} + \pi n$

or $-\frac{\pi}{4} + \pi n$

$\varphi = \pm\frac{2\pi}{3} + 2\pi n$

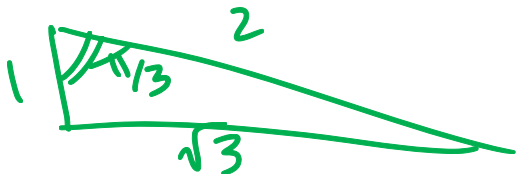
$\frac{x}{3} = \pm\frac{2\pi}{3} + 2\pi n$



$\frac{1}{2}(x+\frac{\pi}{3}) = \pm\frac{\pi}{4} + \pi n$

$x = \pm 2\pi + 6\pi n$

$x = \left(\pm\frac{\pi}{2} - \frac{\pi}{3}\right) + \pi n$



$2y^2 - 3y + 1 = 0$

Practice:

$2\sin^2 x - 3\sin x + 1 = 0$

$2\cos\left(\frac{\pi}{5}(x-3)\right) + 1 = 0.5$

$$\tan^2 2x + 4\tan 2x - 5 = 0$$

$$4\cos^3\left(\frac{\pi}{4}(x+1)\right) = 3\cos\left(\frac{\pi}{4}(x+1)\right)$$

$$\csc^2\left(\frac{3}{5}\left(x-\frac{\pi}{2}\right)\right)=4$$

$$\sec^2\left(\frac{\pi}{12}(x+3)\right)=2$$

$$\frac{2}{3}\sec\left(\frac{\pi}{5x}\right) = 1$$

$$5\cot\left(\frac{x^2}{6}\right) - 3 = 0$$

Practice Problems: Zeros of the practice graphing sheet (when available)