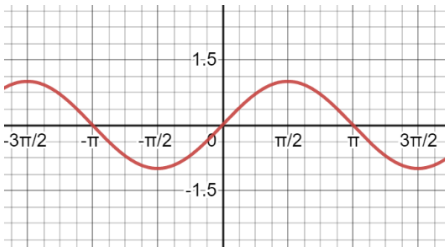
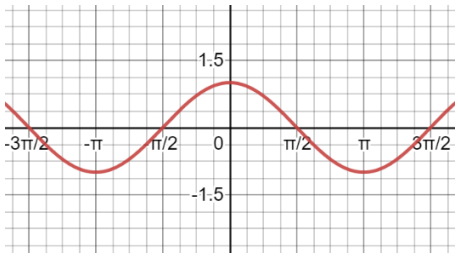
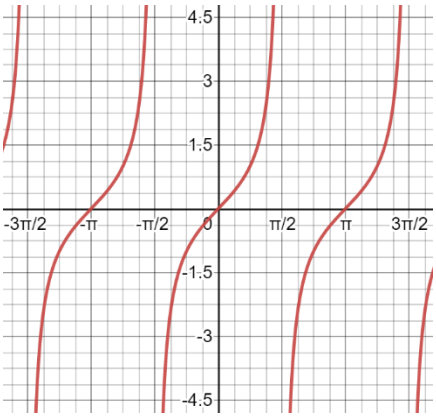
Solving Sinusoidal Functions

KNOW	DO	UNDERSTAND
There are multiple solutions to a trig equation.	Can find the solutions to a trig equation in a given domain. Can use special triangles when appropriate.	*Inverse:* Sine and cosine are not 1-to-1 so the domain must be restricted. Restrictions come so that they take on all values of the range once.

Vocab & Notation

- $\arcsin x$, $\arccos x$, $\arctan x$

Note how the domain gets restricted for the inverse functions:

$f: \mathbb{R} \to [-1, 1]$ $x \mapsto \sin x$	$g: \mathbb{R} \to [-1, 1]$ $x \mapsto \cos x$	$h: \mathbb{R} \backslash \{x \mid \cos x = 0\} \to \mathbb{R}$ $x \mapsto \tan x$
		
Example: $\sin x = 0.8$	**Example**: $\cos x = 0.8$	**Example**: $\tan x = 0.8$

Example (With Calculator) Use algebra to solve the following trig equations:

$$\frac{1}{2}\sin(\pi(x-0.1)\,)=0.2$$

Example (Without Calculator)

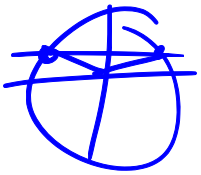
$$\left(\tan^2\left(\frac{1}{2}\left(x+\frac{\pi}{3}\right)\right)-1\right)\left(2\cos\left(\frac{x}{3}\right)+1\right)=0$$

Practice:

$$2\sin^2 x - 3\sin x + 1 = 0$$

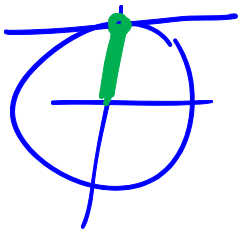
$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = \frac{1}{2}$



$\Rightarrow x = \frac{\pi}{6} + 2\pi n$ or $x = \frac{5\pi}{6} + 2\pi n$

OR $\sin x = 1$



$\Rightarrow x = \frac{\pi}{2} + 2\pi n$, $n \in \mathbb{Z}$

$$2\cos\left(\underbrace{\frac{\pi}{5}(x-3)}_{\theta}\right) + 1 = 0.5$$

$2\cos\theta + 1 = 0.5$

$\cos\theta = -\frac{1}{4}$

$\Rightarrow \theta = \pm 1.823 + 2\pi n = \frac{\pi}{5}(x-3)$

$\Rightarrow x - 3 = \pm 2.902 + 10n$

$x = (3 \pm 2.902) + 10n \quad n \in \mathbb{Z}$

$$\tan^2 2x + 4\tan 2x - 5 = 0$$

$(\tan 2x + 5)(\tan 2x - 1) = 0$

$\tan 2x = -5$ or $\tan 2x = 1$

$2x = -1.373 + \pi n$ or $2x = \frac{\pi}{4} + \pi n$

$\Rightarrow x = -0.687 + \frac{\pi}{2} n$ or

$x = \frac{\pi}{8} + \frac{\pi}{2} n$, $n \in \mathbb{Z}$

$$4\cos^3\left(\underbrace{\frac{\pi}{4}(x+1)}_{\theta}\right) = 3\cos\left(\frac{\pi}{4}(x+1)\right)$$

$\Rightarrow 4\cos^3\theta - 3\cos\theta = 0$

$\cos\theta(4\cos^2\theta - 3) = 0$

$\cos\theta = 0$ or $\cos\theta = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \theta = \pm\frac{\pi}{2} + 2\pi n$ or $\theta = \pm\frac{\pi}{6} + 2\pi n$

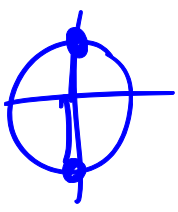
or $\theta = \pm\frac{5\pi}{6} + 2\pi n$

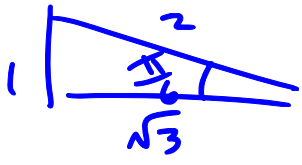
$\Rightarrow \frac{\pi}{4}(x+1) = \pm\frac{\pi}{2}, \pm\frac{\pi}{6}, \pm\frac{5\pi}{6} + 2\pi n$

$\Rightarrow x+1 = \left(\pm 2, \pm\frac{2}{3}, \pm\frac{10}{3}\right) + 8n$

$x = \left\{1, -3, -\frac{1}{3}, -\frac{5}{3}, \frac{7}{3}, -\frac{13}{3}\right\} + 8n$

$n \in \mathbb{Z}$





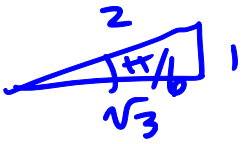
$$\csc^2\left(\underbrace{\frac{3}{5}\left(x-\frac{\pi}{2}\right)}_{\theta}\right)=4$$

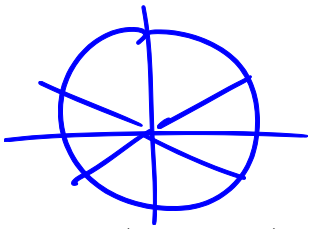
$\sin^2\theta = \frac{1}{4}$ $\Rightarrow \frac{3}{5}\left(x-\frac{\pi}{2}\right) = \pm\frac{\pi}{6} + \pi n$

$\sin\theta = \pm\frac{1}{2}$ $x-\frac{\pi}{2} = \pm\frac{5\pi}{18} + \frac{5}{3}\pi n$

$\theta = \pm\frac{\pi}{6}, \pm\frac{5\pi}{6} + 2\pi n$ $x = \frac{\pi}{2} \pm \frac{5\pi}{18} + \frac{5}{3}\pi n$, $n \in \mathbb{Z}$

$\Rightarrow \theta = \pm\frac{\pi}{6} + \pi n$





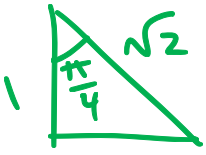
$$\sec^2\left(\underbrace{\frac{\pi}{12}(x+3)}_{\theta}\right)=2$$

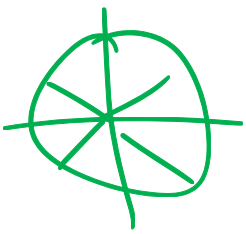
$\cos^2\theta = \frac{1}{2}$ $\Rightarrow \frac{\pi}{12}(x+3) = \frac{\pi}{4} + \frac{\pi}{2}n$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$ $x+3 = 3 + 6n$

$\Rightarrow \theta = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4} + 2\pi n$ $x = 6n$, $n \in \mathbb{Z}$

$\Rightarrow \theta = \frac{\pi}{4} + \frac{\pi}{2}n$





$$\frac{2}{3}\sec\left(\frac{\pi}{5x}\right) = 1$$

$\Rightarrow \frac{2}{3\cos(\theta)} = 1$

$\cos\theta = \frac{2}{3}$

$\theta = \pm 0.841 + 2\pi n$

$\Rightarrow \frac{\pi}{5x} = \pm 0.841 + 2\pi n$

$\Rightarrow \frac{1}{x} = \pm 1.339 + 10n$

$x = \frac{1}{10n \pm 1.339}$ $n \in \mathbb{Z}$

$$5\cot\left(\frac{x^2}{6}\right) - 3 = 0$$

$5\cot\left(\frac{x^2}{6}\right) = 3$

$\tan\left(\frac{x^2}{6}\right) = \frac{5}{3}$

$\frac{x^2}{6} = 1.03 + \pi n$

$x = \pm\sqrt{6.182 + 6\pi n}$ $n \in \mathbb{Z}$

Practice Problems: Zeros of the practice graphing sheet (when available)