Solving Sinusoidal Functions (Intro)

KNOW	DO	UNDERSTAND
There are multiple solutions to a trig equation.	Can find the solutions to a trig equation in a given domain.	*Inverse:* Sine and cosine are not 1-to-1 so the domain must be restricted. Restrictions come so that they take on all values of the range once.

Vocab & Notation

- Inverse trig: $\sin^{-1}(\quad)$; $\arcsin(\quad)$

If $x^2 = 8$ what is x?

$\Rightarrow x = \sqrt{8}$ or $-\sqrt{8}$

★ x^2 even

$f(x) = x^2$ $f^{-1}(x) = \sqrt{x}$

$x \geq 0$

But x^2 is NOT 1-to-1

so we need to find the other solution

phi

So, when we ask: if $\cos\theta = 0.8$ or if $\sin\varphi = 0.8$ or if $\tan\beta$, then what is θ, φ and β? We have the same problem.

$\tan\beta = 0.8$

$\cos^{-1}(\cancel{\cos}\theta = 0.8)$

$\theta = \cos^{-1}(0.8)$

$= 0.64$ or -0.64

$\theta = 0.64 + 2\pi n$, $n \in \mathbb{Z}$

or $-0.64 + 2\pi n$

$\arcsin(\cancel{\sin}\theta = 0.8)$

$\theta = \arcsin 0.8$

$\theta = 0.93, 2.21$

$\Rightarrow \theta = 0.93 + 2\pi n$

or $2.21 + 2\pi n$

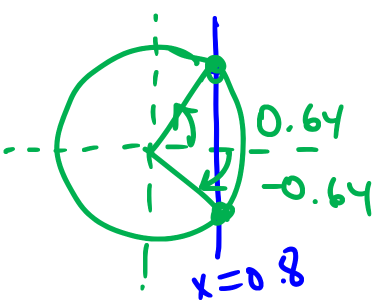
$\arctan(\cancel{\tan}\beta = 0.8)$

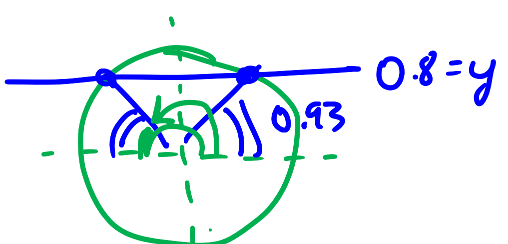
$\beta = \arctan 0.8$

$\beta = 0.67$

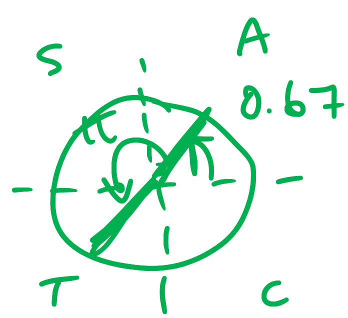
$\beta = 0.67 + \pi n$

When we use the inverse we are only finding one solution. Recognize that there will almost always be a second solution (sometimes three other solutions if we can be positive or negative)





$\pi - 0.93 = 2.21$



Example: Solve for x

$$4\sin^2\Big(\underbrace{\frac{\pi}{2}(x-1)}_{\theta}\Big) = 1$$

$4\sin^2\theta = 1$

$\sin\theta = \pm\frac{1}{2}$

$\Rightarrow \theta = \pm\frac{\pi}{6}, \pm\frac{5\pi}{6}$

$\theta = \pm\frac{\pi}{6} + 2\pi n$ or $\pm\frac{5\pi}{6} + 2\pi n$

$\frac{\pi}{2}(x-1) = \pm\frac{\pi}{6} + 2\pi n$ or $\frac{\pi}{2}(x-1) = \pm\frac{5\pi}{6} + 2\pi n$

$x = 1 \pm \frac{1}{3} + 2\pi n$ or $x = 1 \pm \frac{5}{3} + 2\pi n$, $n \in \mathbb{Z}$

Example: Solve for x

$$\tan^2\left(2\left(x+\frac{\pi}{3}\right)\right) = 5\tan\left(2\left(x+\frac{\pi}{3}\right)\right)$$

$\tan^2\theta - 5\tan\theta = 0 \Rightarrow \tan\theta(\tan\theta - 5) = 0$

$\tan\theta = 0$ or $\tan\theta = 5$

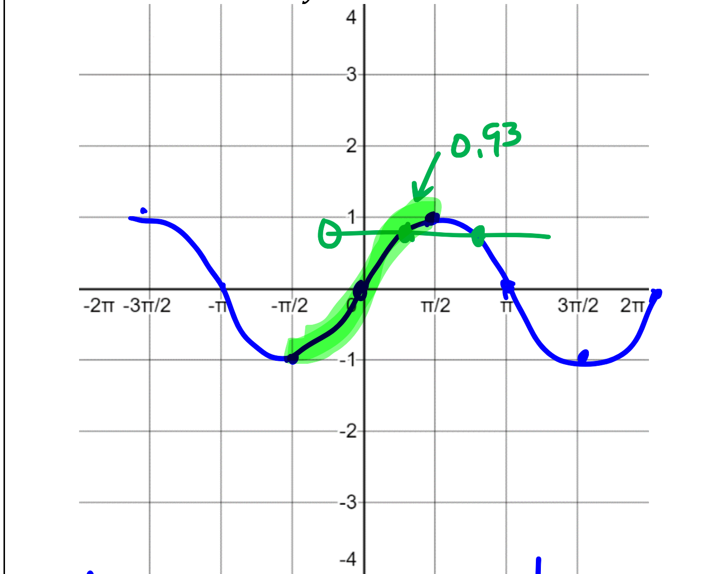
$\Rightarrow \theta = 0 + \pi n$ or $\theta = 1.37 + \pi n$, $n \in \mathbb{Z}$

$2\left(x + \frac{\pi}{3}\right) = \pi n$ or $2\left(x + \frac{\pi}{3}\right) = 1.37 + \pi n$

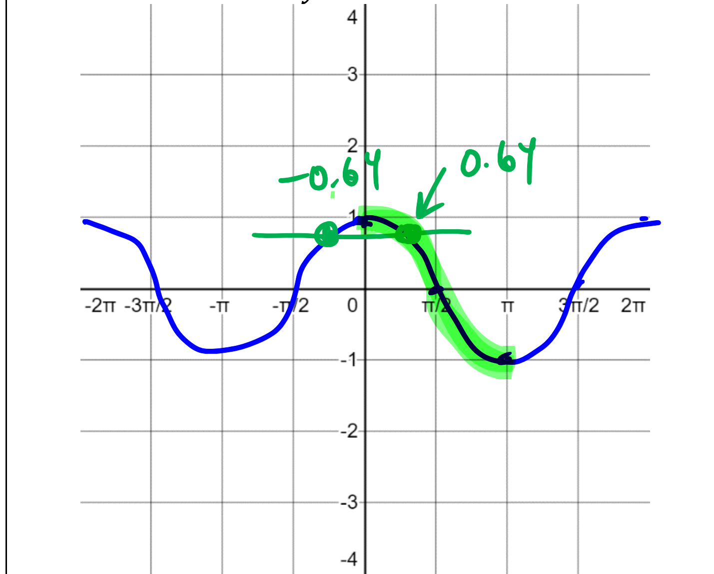
$x = -\frac{\pi}{3} + \frac{\pi}{2}n$ or $x = -0.36 + \frac{\pi}{2}n$ $n \in \mathbb{Z}$

Practice Problems: 4.4 page 211 – 213 Examples #1-3, Questions # 3-5, 7

$y = \sin x$

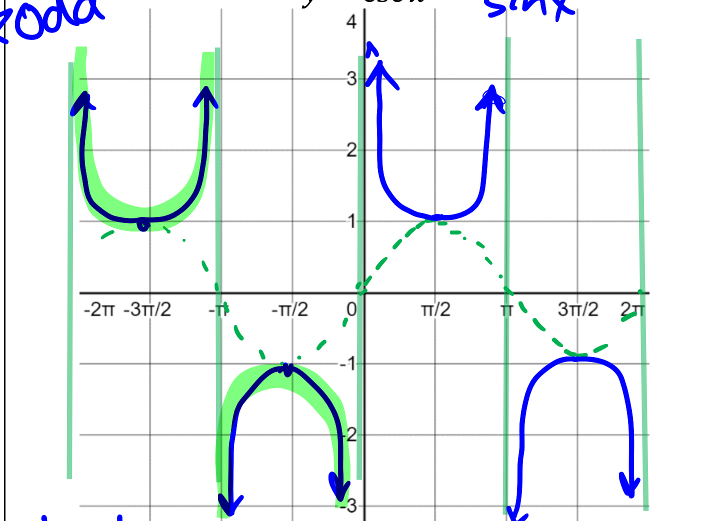


$y = \cos x$



★ odd

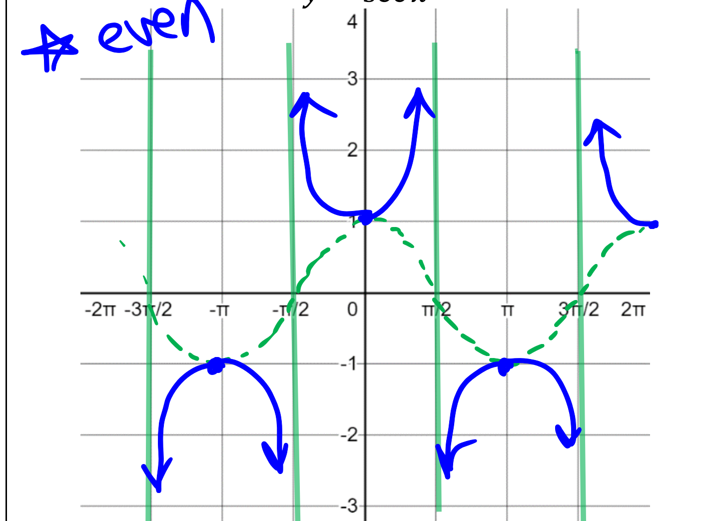
$y = \csc x = \frac{1}{\sin x}$



$|y| \geq 1$ $T = 2\pi$

★ even

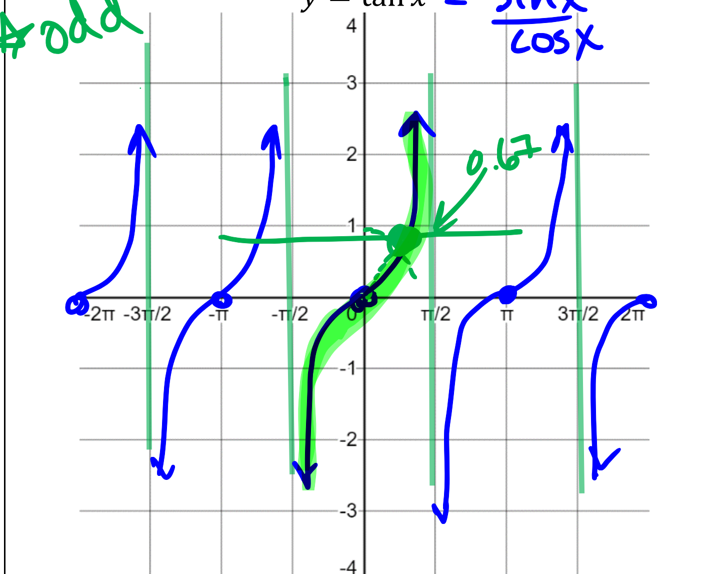
$y = \sec x$



$|y| \geq 1$ $T = 2\pi$

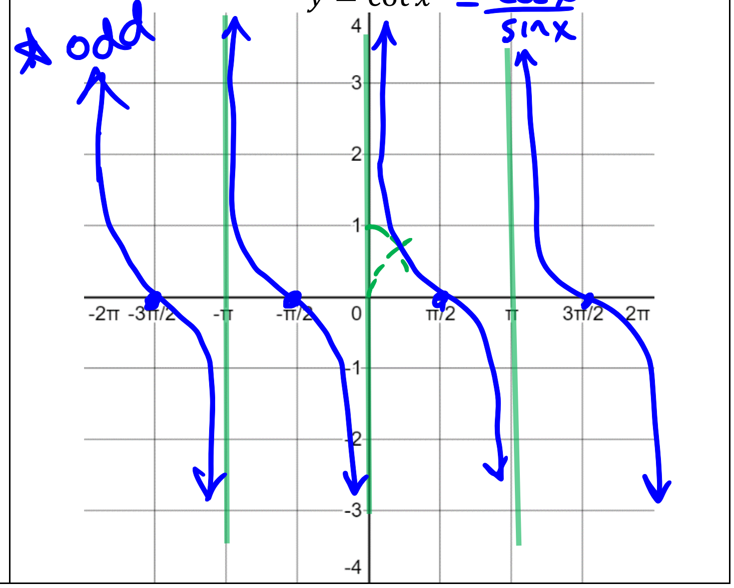
★ odd

$y = \tan x = \frac{\sin x}{\cos x}$



★ odd

$y = \cot x = \frac{\cos x}{\sin x}$



$y \in \mathbb{R}$ $T = \pi$ ★

$y \in \mathbb{R}$ $T = \pi$