Solving Sinusoidal Functions (Intro)

KNOW	DO	UNDERSTAND
There are multiple solutions to a trig equation.	Can find the solutions to a trig equation in a given domain.	*Inverse:* Sine and cosine are not 1-to-1 so the domain must be restricted. Restrictions come so that they take on all values of the range once.

Vocab & Notation

- Inverse trig: $\sin^{-1}(\quad)$; $\arcsin(\quad)$

$f(-x) = f(x)$

If $x^2 = 8$ what is x?

$x = \sqrt{8}$ or $-\sqrt{8}$

$f(x) = x^2$, $x \geq 0$ $\Rightarrow$ $f^{-1}(x) = \sqrt{x}$

Because x^2 is not 1-to-1 we must restrict domain to get an inverse that is a function.

If x_0 is a solution $\Rightarrow$ $-x_0$ is too. B/c x^2 is even

So, when we ask: if $\cos\theta = 0.8$ or if $\sin\varphi = 0.8$ or if $\tan\beta$, then what is θ, φ and β? We have the same problem.

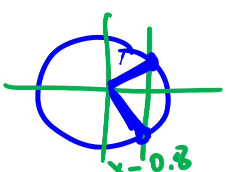
$\tan\beta = 0.8$

$\cancel{\cos}^{-1}(\cancel{\cos}\,\theta = 0.8)$

$\theta = \cos^{-1}(0.8)$

$\theta = 0.64\ldots + 2\pi n,\ n \in \mathbb{Z}$

or $-0.64 + 2\pi n$

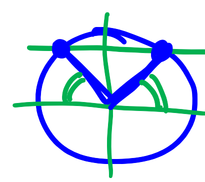


$\cancel{\sin}^{-1}(\cancel{\sin}\,\varphi = 0.8)$

$\varphi = \sin^{-1}(0.8)$

$\varphi = 0.93$

or $\pi - 0.93$



$\varphi = 0.93 + 2\pi n,\ n \in \mathbb{Z}$

or $2.21 + 2\pi n$

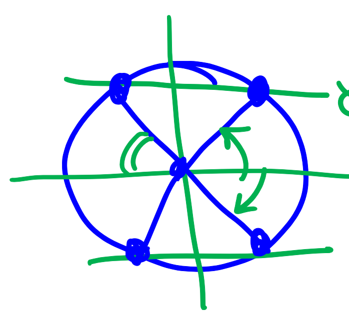
When we use the inverse we are only finding one solution. Recognize that there will almost always be a second solution (sometimes three other solutions if we can be positive or negative)

$\sqrt{\sin^2\theta} = 0.8$

$\cancel{\arcsin}(\cancel{\sin}\,\theta = 0.89$ or $-0.89)$

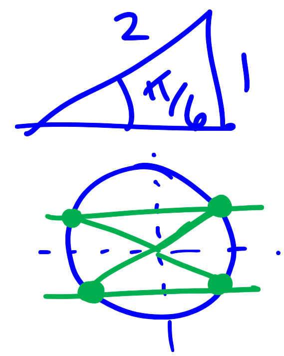
$\theta = \pm 1.097 + 2\pi n,\ n \in \mathbb{Z}$

or $\theta = \pm(\pi - 1.097) + 2\pi n$



Example: Solve for x

$$4\sin^2\left(\frac{\pi}{2}(x-1)\right) = 1$$



$\Rightarrow 4\sin^2\theta = 1 \quad \Rightarrow \sin\theta = \pm\frac{1}{2}$

$\Rightarrow \theta = \pm\frac{\pi}{6} + 2\pi n$ or $\theta = \pm\frac{5\pi}{6} + 2\pi n$

$\Rightarrow \frac{\pi}{2}(x-1) = \pm\frac{\pi}{6} + 2\pi n$ or $\frac{\pi}{2}(x-1) = \pm\frac{5\pi}{6} + 2\pi n$

$(x-1) = \pm\frac{1}{3} + 4n$ or $(x-1) = \frac{\pm 5}{3} + 4n$

$x = \left(1 \pm \frac{1}{3}\right) + 4n$ or $x = \left(1 \pm 5/3\right) + 4n \quad , n \in \mathbb{Z}$

$x = \left\{\frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \frac{-2}{3}\right\} + 4n$

Example: Solve for x

$$\tan^2\left(2\left(x+\frac{\pi}{3}\right)\right) = 5\tan\left(2\left(x+\frac{\pi}{3}\right)\right)$$

$\Rightarrow \tan^2\theta = 5\tan\theta$

$\Rightarrow \tan^2\theta - 5\tan\theta = 0 \qquad \tan\theta(\tan\theta - 5) = 0$

$\Rightarrow \tan\theta = 0$ or $\tan\theta = 5$

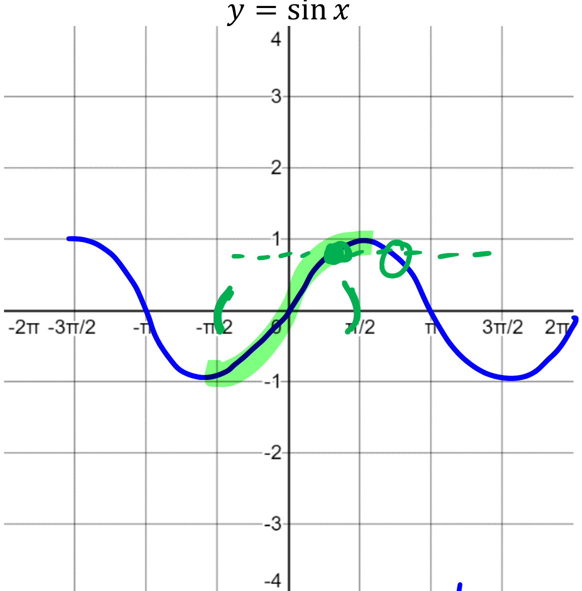
$\Rightarrow \theta = 0 + \pi n$ or $\theta = 1.37 + \pi n \quad , n \in \mathbb{Z}$

$2\left(x + \frac{\pi}{3}\right) = \pi n$ or $2\left(x + \frac{\pi}{3}\right) = 1.37 + \pi n$

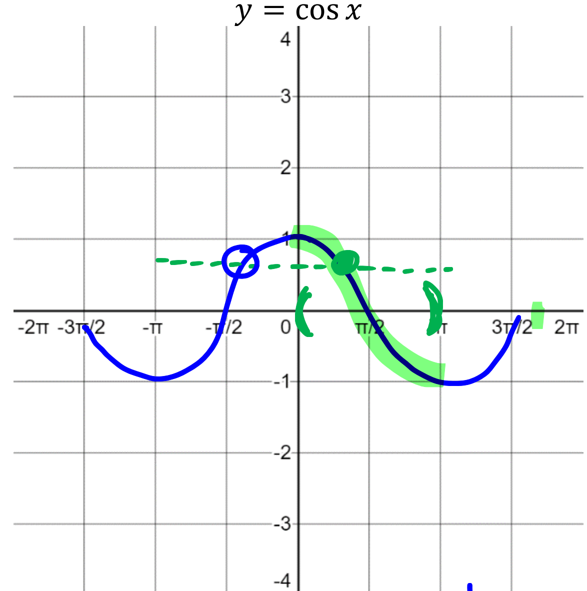
$\Rightarrow x = \frac{-\pi}{3} + \frac{\pi n}{2}$ or $x = -0.36 + \frac{\pi n}{2} \quad n \in \mathbb{Z}$

Practice Problems: 4.4 page 211 – 213 Examples #1-3, Questions # 3-5, 7

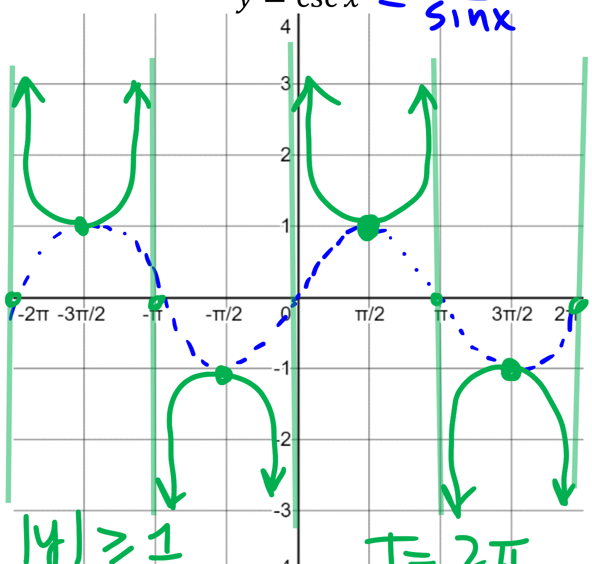
$y = \sin x$



$y = \cos x$

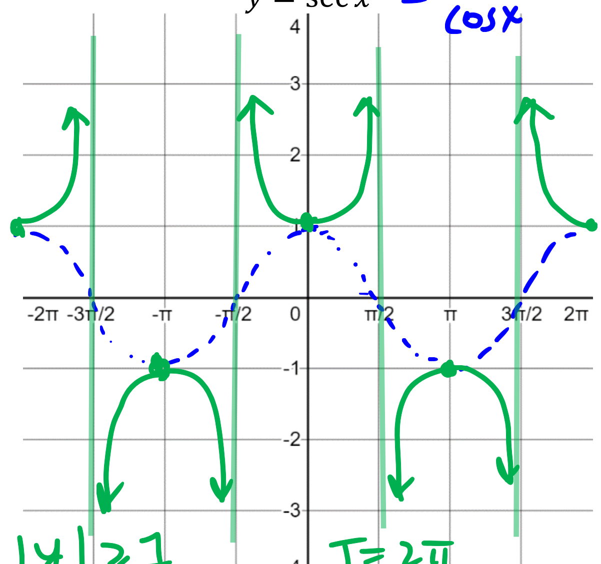


$y = \csc x = \frac{1}{\sin x}$



$|y| \geq 1$ $T = 2\pi$

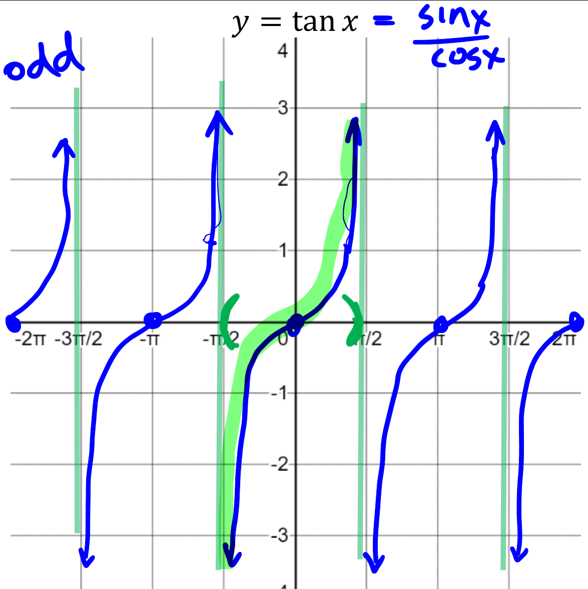
$y = \sec x = \frac{1}{\cos x}$



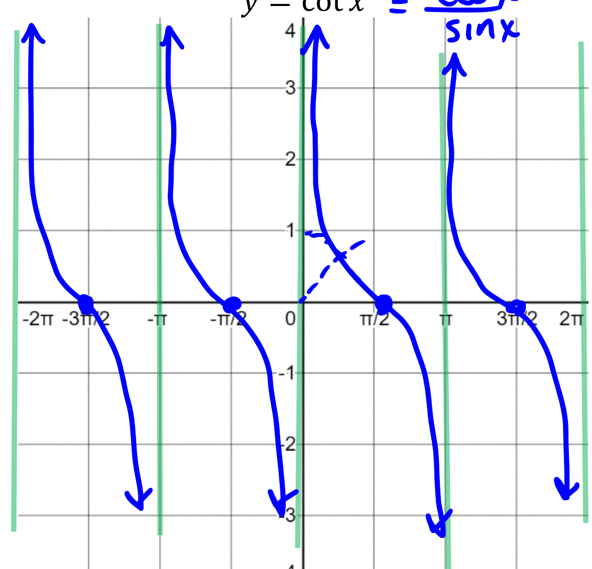
$|y| \geq 1$ $T = 2\pi$

★odd

$y = \tan x = \frac{\sin x}{\cos x}$



$y = \cot x = \frac{\cos x}{\sin x}$



$y \in \mathbb{R}$ $T = \pi$

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