Steady States and Natural Growth Solutions

1.

a. $y = 10e^{\frac{t}{3}}$

b. Steady state of $z = \frac{2}{3}$ so let $\Gamma = z - \frac{2}{3}$

$$\frac{dz}{dx} = 2 - 3\left(\Gamma + \frac{2}{3}\right) = -3\Gamma = \frac{d\Gamma}{dx} \Rightarrow \Gamma = Ce^{-3x} = z - \frac{2}{3}$$

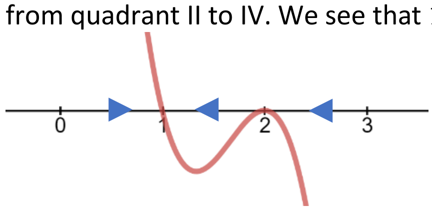
Set $x = 0$ and $z = -2$ to solve $C = -\frac{8}{3}$

$$z = -\frac{8}{3}e^{-3x} + \frac{2}{3}$$

2. We are looking for what p values make $\frac{dp}{dq} = 0$ as $q \to \infty$. For us $(3-q)(4-q) \to +\infty$ so the differential equation we want is

$$\frac{dp}{dq} = +(1-p)(2-p)^2$$

Which has steady states at $p = 1$ and $p = 2$. And the graph of $\frac{dp}{dq}$ will be a cubic that is going from quadrant II to IV. We see that 1 is a stable steady state and 2 is semi-stable.



3. We have that $k = 0.023$ and the unstable steady state is \$0

$$\frac{dc}{dt} = 0.023c \Rightarrow c = Ce^{0.023t}, \qquad c(0) = 3 = C$$
$$c(t) = 3e^{0.023t}$$
$$c(20) = \$4.75$$

4. There is a stable steady state at $T = 20$ seconds.

$$\frac{dT}{dt} = k(T - 20), \qquad T(0) = 300, \qquad T(12) = 45$$

Let $\Gamma = T - 20$ and so

$$\frac{dT}{dt} = k\Gamma = \frac{d\Gamma}{dt} \Rightarrow \Gamma = Ce^{kt} = T - 20$$

We find $C = 280$ and we can solve for k

$$k = \ln\left(\frac{25}{280}\right) \cdot \frac{1}{12} = -0.201$$

$$T(t) = 280e^{-0.201t} + 20$$
$$T(x) = 30$$
$$\Rightarrow x = \ln\left(\frac{1}{28}\right) \cdot \frac{1}{-0.201} = 16.6 \text{ weeks}$$