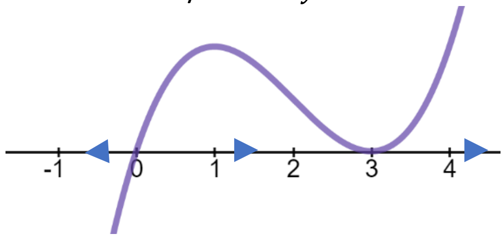
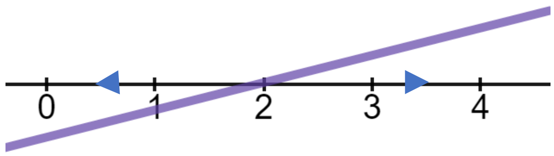
SOLUTIONS:

1.

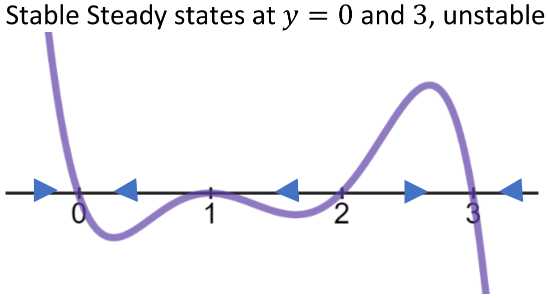
a. Unstable steady state at $y = 0$ and semi-stable at $y = 3$



b. Unstable steady state when $y = 2$. Let $t \to \infty$ so $\frac{dy}{dt} = +(y - 2)$

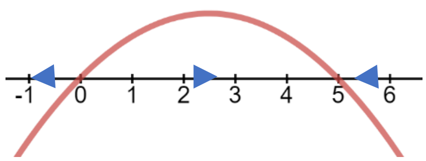


c. Stable Steady states at $y = 0$ and 3, unstable steady state at $y = 2$, and semi stable steady state at $y = 1$.

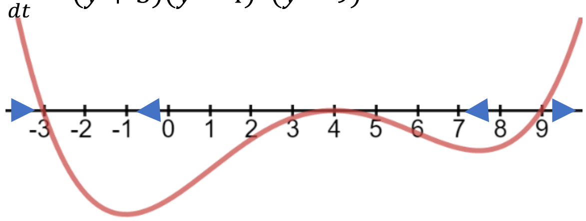


2.

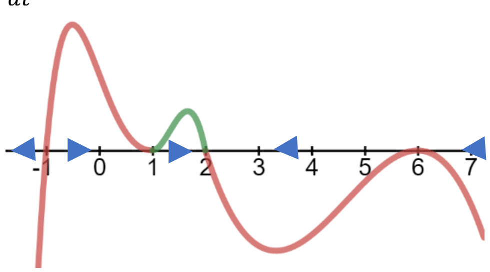
a. $\frac{dy}{dt} = -y(y - 5)$



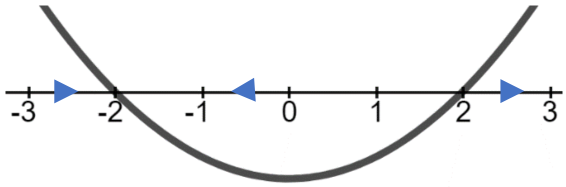
b. $\frac{dy}{dt} = (y + 3)(y - 4)^2(y - 9)$



c. $\frac{dy}{dt} = -(y + 1)(y - 1)^2(y - 2)(y - 6)^2$



3. Because if you make one steady state stable it must approach it from both sides which means it has to move away from another point. Therefore, the other point must have some instability and they both can't be stable. A similar argument works for if one is unstable. Also, you could make an exhaustive list as there are only 8 ways you can organize your arrows $(>,>,>)$; $(>,>,<)$; $(>,<,>)$; $(>,<,<)$, etc and you can see that in no case will there be two stable or unstable steady states.
4. Steady states when $M = \pm 2$. We let $r \to \infty$ and so $\frac{dM}{dr} = +(M^2 - 4)$ and we see that $M = -2$ is stable while $M = 2$ is unstable.



5.
   a. There is a semi-stable steady state at $F = -4$, an unstable steady state at $F = 3$, and there is a stable steady state when $F = 0$
$$\Rightarrow \frac{dF}{db} = F(F+4)^2(F-3)$$
   b. There is an unstable steady state at $z = 2$ and a stable steady state at $z = -1$. Note that for some $k > 0$ we have that when $x = k$ the slope is 0. Ignoring the x value we can make the differential
$$\frac{dz}{dx} = +(z-2)(z+1)$$
That has the right steady states and we just want our term with x in it to stay positive as $x \to \infty$. So…
$$\frac{dz}{dx} = (x-k)(z-2)(z+1)$$