The Tangent as a Limit

Goal: • Can determine the slope of a tangent line using limits as x approaches a point c AND as the distance between points, h, approaches 0.
Terminology: • None
Reminders: • Quiz Monday Oct 7 • Test Friday Oct 11 • Get evidence up to date!

Review: Determine the slope of the tangent line of $y = x^3$ at the point $x = 3$ to 2 decimal places.

near $x=3$ try $x = 3.001$ and 2.999

$$\frac{\Delta y}{\Delta x} = \frac{3.001^3 - 3^3}{0.001} = 27.009$$

$$\text{OR} = \frac{2.999^3 - 3^3}{-0.001} = 26.991$$

27.00

On the board: Formalize your ideas above to find the slope of the function $f(x)$ at the point $x = c$.

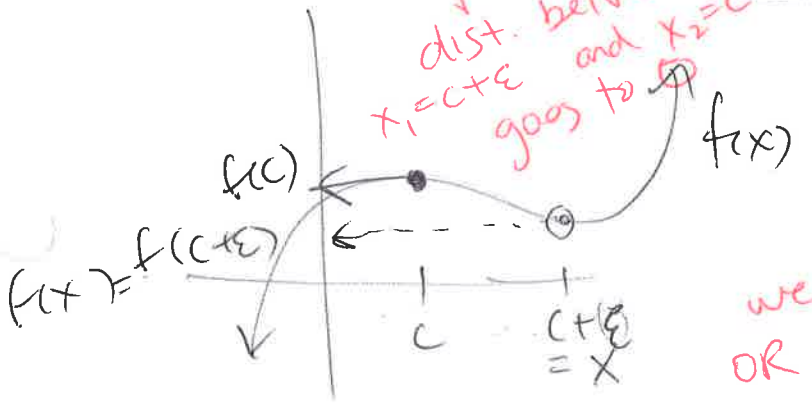
$$\frac{f(c+0.001) - f(c)}{0.001}$$

ε close to 0 $\Rightarrow \varepsilon \to 0$

$$= \lim_{\varepsilon \to 0} \frac{f(c+\varepsilon) - f(c)}{\varepsilon} = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

dist. between $x_1 = c+\varepsilon$ and $x_2 = c$ goes to 0

$x_1 = x$ and $x_2 = c$ to move together



well $\varepsilon \to 0$
OR $x \to c$

Example: Determine the exact slope of the tangent line of the function $f(x) = 9 - x^2$ at the point $x = 3$.

$$\lim_{\varepsilon \to 0} \frac{f(c+\varepsilon) - f(c)}{\varepsilon} = \lim_{\varepsilon \to 0^+} \frac{9 - (c+\varepsilon)^2 - (9 - c^2)}{\varepsilon}$$

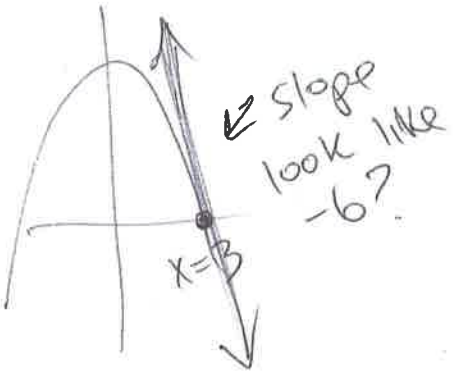
$c = 3$

$$= \lim_{\varepsilon \to 0} \frac{9 - (3+\varepsilon)^2 - 0}{\varepsilon}$$

$$= \lim_{\varepsilon \to 0} \frac{9 - (9 + 2\cdot 3\varepsilon + \varepsilon^2)}{\varepsilon}$$

$$= \lim_{\varepsilon \to 0} -6 - \varepsilon = -6$$

let $\varepsilon = 0$



Example: Determine the exact slope of the tangent line of the function $g(x) = \frac{2x+1}{x}$ at the point $x = c$.

$$\lim_{x \to c} \frac{g(x) - g(c)}{x - c} = \lim_{x \to c} \left(\frac{2x+1}{x} - \frac{2c+1}{c} \right) \div (x - c)$$

want to cancel

$$= \lim_{x \to c} \frac{2xc + c - 2cx - x}{cx(x-c)} \quad (-1)$$

$$= \lim_{x \to c} \frac{-1}{cx} = \boxed{-\frac{1}{c^2}}$$

let $c = x$

Practice Problems: 1.4: # 1, 2, 7, 8,

9, 10