The Tangent Problem

Goal: • Can use determine the slope of a curve at a point using secant lines and refining the distance between points.
Terminology: • Tangent • Secant

We were given data on the water levels of the Fraser River for September 7, 2019 and were asked to determine when the water levels were changing the quickest. That is, we want to find the point that maximizes the value of depth (m) per second (s).

$\max \left| \frac{d(t)}{t} \right|$ uints $\frac{m}{s}$

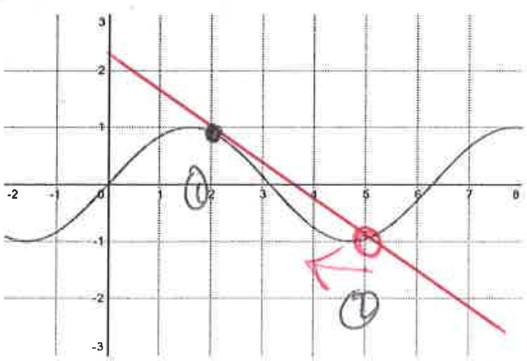
Tangent around the point

This means we need to maximize slope and is equivalent to finding the steepest **tangent line.**

will touch the curve once → has the same slope

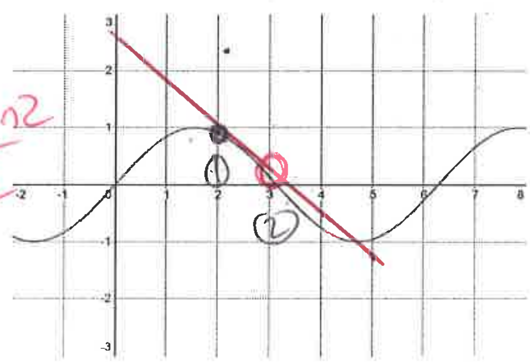
We do this by approximating the value using **secant lines** that look like a tangent line.

Example: What is the slope of $f(x) = \sin x$ at the point $x = 2$?



★ if we had one point

$\text{slope} = \frac{\sin 2 - \sin 2}{2-2} = \frac{0}{0}$?



$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{\sin 5 - \sin 2}{5-2} = -0.623$

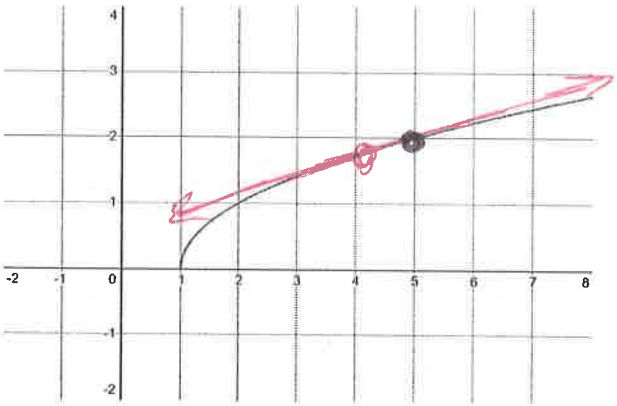
$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{\sin 3 - \sin 2}{3-2} = -0.768$

want red dot, ○, to get close to $2 = x$

$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{\sin 2.1 - \sin 2}{2.1-2} = -0.461$

try $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{\sin 2.001 - \sin 2}{2.001-2} = -0.417$

Practice: Determine the slope of $g(x) = \sqrt{x-1}$ at $x = 5$.



let $x = 4.999$ and 5.001

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{g(5.001) - g(5)}{5.001 - 5} = \frac{\sqrt{4.001} - 2}{0.001} = 0.24999$$

OR

$$\frac{g(4.999) - g(5)}{4.999 - 5} = \frac{\sqrt{3.999} - 2}{-0.001} = 0.25000$$

$$\boxed{\text{Slope} = 0.25}$$

Group: Write an expression and solve for the slope of $h(x) = \frac{1}{x+2}$ at the point $x = x_0$

Practice Problems: 1.1 # 4, 5, 7-10

11, 12