Limits of Trig Functions

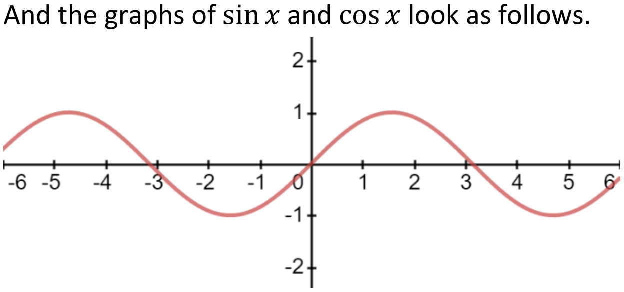
Goal: • Understands the importance of the radian when considering trig rates of change.
Terminology: • Radian

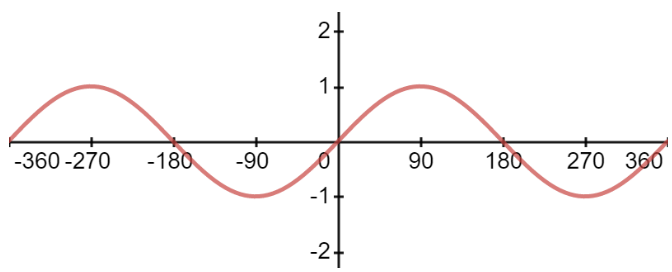
Discussion: What is a degree? What is a radian?

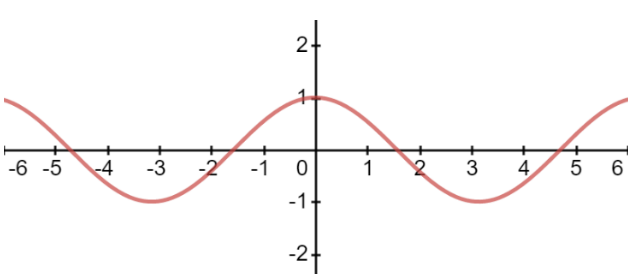
Recall some important trig identities:

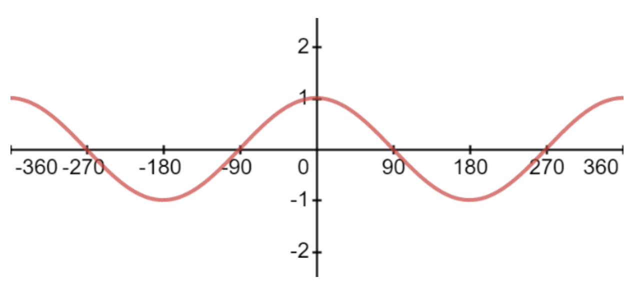
$\sin^2 x + \cos^2 x = 1$	$\tan x = \dfrac{\sin x}{\cos x}$	$\dfrac{1}{\sin x} = \csc x$
$\sin(-x) = -\sin x$	$\dfrac{1}{\tan x} = \cot x$	$\dfrac{1}{\cos x} = \sec x$
$\cos(-x) = \cos x$	$\sin(A+B) = \sin A \cos B + \sin B \cos A$	$\cos(A+B) = \cos A \cos B - \sin A \sin B$

And the graphs of $\sin x$ and $\cos x$ look as follows.









We want to study the calculus of trig functions which begins with limits. No matter the unit of measurement we have that

$$\lim_{x\to 0} \sin x = 0$$

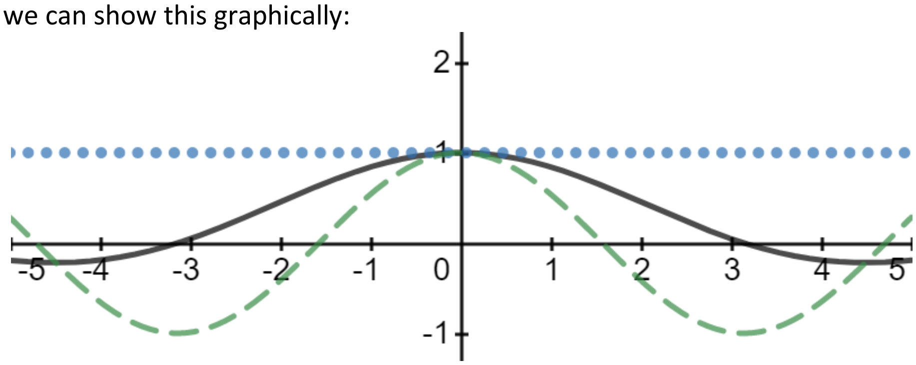
It may seem like it doesn't matter between radian or degree, but for almost every other value of $c \in \mathbb{R}$ we have that

$$\lim_{x\to c\text{ (rad)}} \sin x \neq \lim_{x\to c\text{ (deg)}} \sin x$$

So very clearly the way we measure the angle is important and makes a difference. The limit we are going to investigate is

$$\lim_{x\to 0} \frac{\sin x}{x}$$

Notice that around $x = 0$, when using radians, $\sin x \approx x$, so we should expect the limit to be around 1. Typically this is shown by "squeezing" the function $\frac{\sin x}{x}$ between the functions $y = 1$ and $y = \cos x$ using geometry (whose limits are both 1 as $x \to 0$). But we can show this graphically:



$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

Note that this implies $\lim_{x\to 0} \frac{x}{\sin x} = 1$ too.

Example: Evaluate the limit

$$\lim_{x\to 0} \frac{\sin 3x}{x^2 - 3x}$$

Practice: Evaluate the limit

$$\lim_{x\to 0} \frac{\sin Ax}{Bx}$$

Practice: Evaluate the limit

$$\lim_{x \to 0} \frac{\sin 2x}{\tan 5x}$$

Example: Evaluate the limit

$$\lim_{x \to 0} \frac{1 - \cos x}{x}$$

Practice: Evaluate the limit

$$\lim_{x \to 0} \frac{1 - \sec x}{\sin x}$$

Practice Problems: 7.1 # 1-38 (what you need)

40-44