Trig Identity Proof

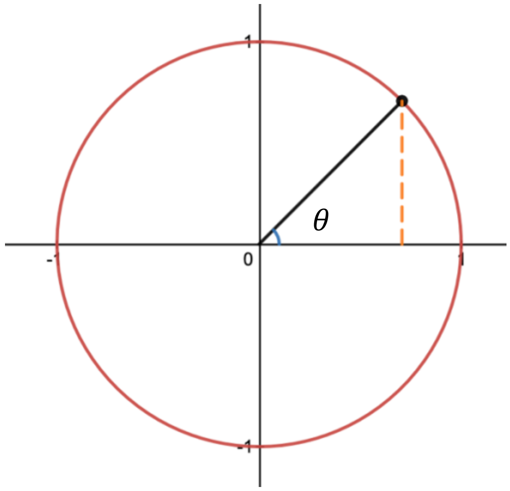
Pythagorean Identities

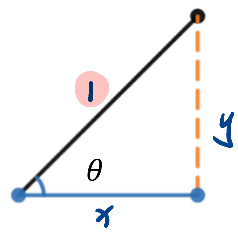
- $cos^2x + sin^2x = 1$
- $tan^2x + 1 = sec^2x$
- $cot^2x + 1 = csc^2x$

} derived from $\tan x = \frac{\sin x}{\cos x}$

Proof:

Derived from coordinates on a unit circle:





Step 1:

Show that $x = \cos\theta, y = \sin\theta$: → SOA CAH TOA

$\cos\theta = \frac{x}{1} = x$ $\quad \sin\theta = \frac{y}{1} = y$

Step 2:

Substitute into the equation based on **Pythagorean Theorem** $(x^2 + y^2 = 1)$:

$x^2 + y^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$

$\Rightarrow$

$1 + \left(\frac{\sin^2\theta}{\cos^2\theta}\right) = \frac{1}{\cos^2\theta}$

$1 + \tan^2\theta = \sec^2\theta$

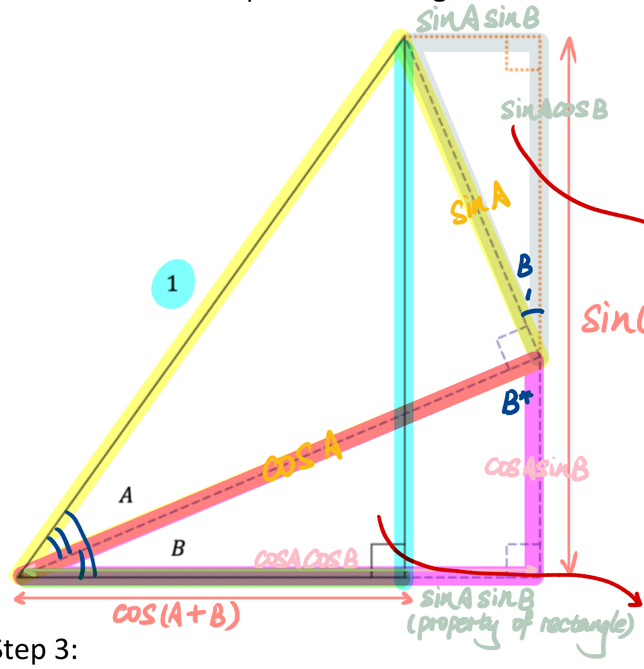
$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$\cot^2\theta + 1 = \csc^2\theta$

Sum of Angles

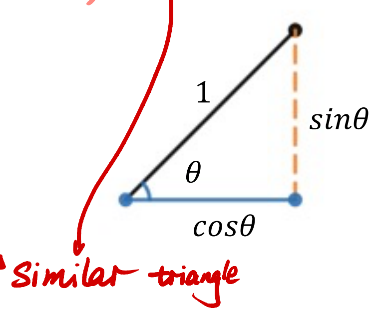
- $\sin(A + B) = \sin A \cos B + \sin B \cos A$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$

Proof:
Derived from a compound of triangles:



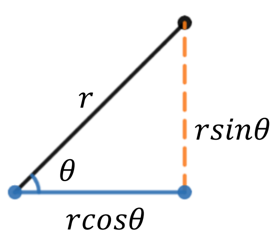
Step 1:
Identify the triangles we need to use:

Step 2:
Find the sides opposite and adjacent to $\angle A + B$ and $\angle A$:

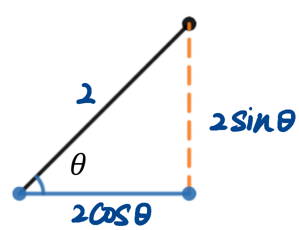


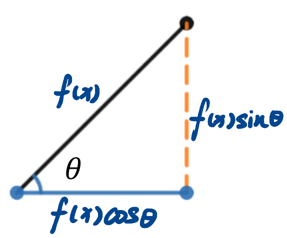
Step 3:
Find the sides opposite and adjacent to $\angle B$ based on $\cos A$:

Based on the previous example, we can generalize the relationship into:



In this case:





Step 4:
Find the angle equal to $\angle B$ based on similar triangle:

$\because \angle B + \angle B^* + \frac{\pi}{2} = \pi$

$\angle 1 + \angle B^* + \frac{\pi}{2} = \pi$

$\therefore \angle B = \angle 1$

Step 5:
Use the same method as step 3 to find the opposite and adjacent sides of $\sin A$:

Step 6: Conclusion

$\therefore \sin(A+B) = \sin A \cos B + \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

Double Angle

- $\sin(2A) = 2\sin A\cos A$
- $\cos(2A) = cos^2 A - sin^2 A$

Proof:

Derived from the sum of angles identity by letting $B = A$:

$\sin(2A) = \sin(A + A) = \sin A\cos A + \sin A\cos B$
$= 2\sin A\cos A$

$\cos(2A) = \cos(A + A) = \cos A \cdot \cos A - \sin A\sin A$
$= \cos^2 A - \sin^2 A$

Note that $\cos 2A = 1 - 2sin^2 A = 2\,cos^2 A - 1$ by using **Pythagorean Identity**:

$\cos(2A) = \cos^2 A - \sin^2 A = 1 - \sin^2 A - \sin^2 A$
$= 1 - 2\sin^2 A$

$1 - 2\sin^2 A = \cos(2A)$

$2\sin^2 A = -\cos(2A) + 1$

$\sin^2 A = \frac{1 - \cos(2A)}{2}$

$\cos(2A) = \cos^2 A - \sin^2 A = \cos^2 A - (1 - \cos^2 A)$
$= 2\cos^2 A - 1$

$2\cos^2 A - 1 = \cos(2A)$

$2\cos^2 A = 1 + \cos(2A)$

$\cos^2 A = \frac{1 + \cos(2A)}{2}$