Trig Modelling Practice

The following problems are snippets from the textbook with the core function erased. Determine the general equation for them, use the equation to predict a value, and use the equation to solve a problem.

1.
   a. A point on saw blade experiences motion around a circle with radius r. It makes n rotations per second. The blade sits so that the center is d units below the top of the table. Determine an equation for the height of the point at time t.

$T = \frac{1}{n}$ sec   midline is $-d$ amp $= r$

$b = 2\pi n$

$$H(t) = r\cos(2\pi n \cdot t) - d$$

t in seconds

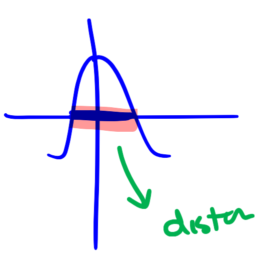
   b. If the radius is 10 cm and it sits 8 cm below the top of the table, ~~how long~~ what % in one rotation will the point be above the table?

$r = 10$ $d = 8$ $H(t) > 0$

$10\cos(2\pi n\, t) - 8 > 0 \Rightarrow \cos\theta = 4/5$

$\Rightarrow \theta = \pm 0.643 + 2\pi N = 2\pi n\, t \quad , N \in \mathbb{Z}$

$$\pm\frac{0.102}{n} + \frac{N}{n} = t$$



distance above 0 is $\frac{0.204}{n}$ sec, one period is $\frac{1}{n}$ seconds

$\Rightarrow$ % above 0 is $\dfrac{\frac{0.204}{n}}{\frac{1}{n}} = 20.4\%$

2.

a. A satellite follows a sinusoidal path over the Earth in orbit. It takes the satellite m minutes to orbit the Earth. On one side of the Earth, it reaches a maximum height of h_1 (km) and on the opposite side is reaches a min height of h_0 (km). At t_0 minutes after noon, the satellite is at the min height. Determine an equation for the height of the satellite at time t.

$T = m$ (min) $\quad H_{max} = h_1 \quad H_{min} = h_0 \quad$; t_0 after 12pm @ h_0

$b = \frac{2\pi}{m}$

$$H(t) = \frac{h_0 - h_1}{2} \cos\left(\frac{2\pi}{m}(t - t_0)\right) + \frac{h_0 + h_1}{2}$$

t is time after noon in min $\quad t=0$ is 12pm

b. If $m = 200$ minutes, $h_1 = 300$ km and $h_0 = 220$ km, and at 12:47 pm the satellite is at the min height, determine the height of the satellite at 5:10 pm.

$t_0 = 47 \qquad t = 5 \cdot 60 + 10 = 310$

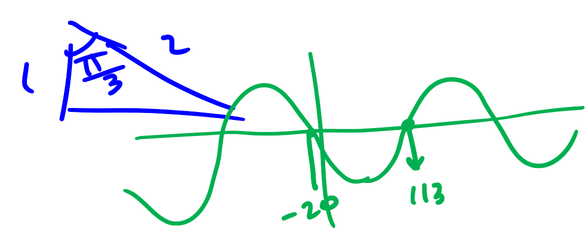
$$H(t) = -40 \cos\left(\frac{\pi}{100}(t - 47)\right) + 260$$

$\Rightarrow H(310) = 275.9$ km

c. Determine the intervals of time from midnight to 6:00 am of that day that the satellite was more then 280 km above the Earth.

$$H(t) = 280 = -40 \cos\left(\frac{\pi}{100}(t - 47)\right) + 260$$

$\cos \frac{\pi}{100}(t-47) = -\frac{1}{2} \quad \Rightarrow \quad \frac{\pi}{100}(t-47) = \pm\frac{2\pi}{3} + 2\pi n$

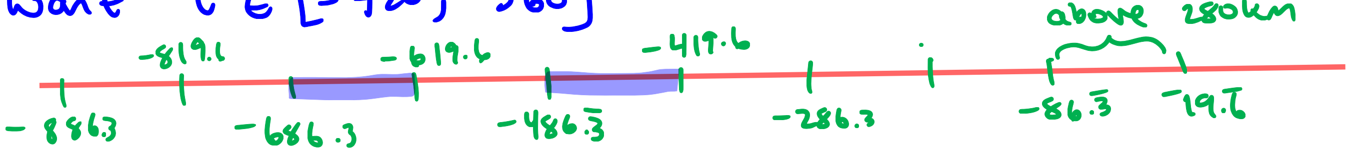


$t - 47 = \pm\frac{200}{3} + 200n$

$t = 113.\overline{6}$ or $-19.\overline{6} + 200n$

starts $\qquad$ end.

want $t \in [-720, -360]$



Above 280km between 12:34 am – 1:40 am and 3:54 am – 5:00 am

3.

a. The population of foxes in a region cycles from a minimum P_0 to maximum P_1 during a m month period (that is from P_0 to P_1 in m months). The population starts at P_0 on the first of month m_0. Determine an equation for the population of foxes at time t in months.

$$P(t) = \frac{P_1 - P_0}{2} \cos\left(\frac{\pi}{m}(t - m_0)\right) + \frac{P_1 + P_0}{2}$$

half period m $\Rightarrow T = 2m$ $b = \frac{\pi}{m}$

time t in months $t = 1 \equiv$ Jan 1st $t = 0 \equiv$ Dec 1st.

b. If $P_0 = 600$ and $P_1 = 1600$, $m = 12$ months, and m_0 is March 2020, determine the population of foxes on June 28th, 2021.

$$P(t) = 500 \cos\left(\frac{\pi}{12}(t - 3)\right) + 1100$$

$P(18.93) = 842$ foxes

June 28th, 2021 $\Rightarrow t = 18.93$

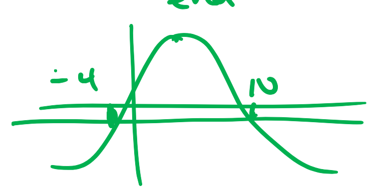
c. Determine the approximate dates between Jan 1, 2020 to December 31, 2024, the population of foxes is greater than 1000.

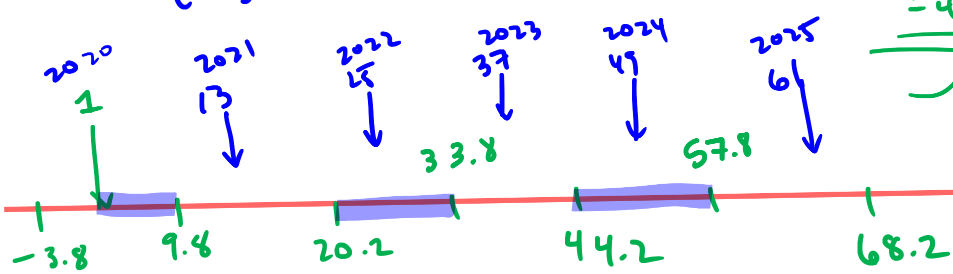
$t \in [1, 61)$ 5 years

$$P(t) = 1000 = 500 \cos\left(\frac{\pi}{12}(t - 3)\right) + 1100$$

$\frac{\pi}{12}(t-3) = \pm 1.77 + 2\pi n \Rightarrow t = 9.77$ (end) or $-3.77 + 24n$ (start)

$t - 3 = \pm 6.77 + 24n$





$P(t) \geq 1000$ from Jan 1, 2020 – Sep 23, 2020 and

Aug 7, 2021 – Sep 23, 2022 and

Aug 7, 2023 – Sep 23, 2024

4.

a. The altitude of the Sun follows a sinusoidal path. The maximum altitude it reaches is θ_1 degrees above the horizon at time t_1 (hours). The lowest it reaches is θ_2 degrees below the horizon at time t_2 (hours). Determine an equation for the height of the Sun as a function of time t.

$T = |t_2 - t_1| \cdot 2$

$b = \frac{\pi}{|t_2 - t_1|}$

t is in hours. $t=0$ is midnight

$$A(t) = \frac{\theta_1 - \theta_2}{2} \cos\left(\frac{\pi}{|t_2 - t_1|}(t - t_1)\right) + \frac{\theta_1 + \theta_2}{2}$$

b. If $\theta_1 = 38°$ at 1:20 pm on March 14, 2021 and $\theta_2 = -42°$ at 1:20 am on Match 15, 2021. Then what was the height of the Sun at 10 am on March 15?

$t=0$ is midnight on mar 15 $\Rightarrow$ $t_2 = 1.\overline{3}$

$t_1 = -10.\overline{6}$

$$A(t) = 40 \cos\left(\frac{\pi}{12}(t + 10.\overline{6})\right) - 2$$

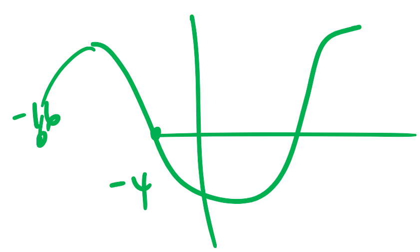
$$A(10) = 23.7°$$

c. Determine the time of sunrise and sunset on March 15, 2021.

$A(t) = 0 = 40 \cos\left(\frac{\pi}{12}(t + 10.\overline{6})\right) - 2$

$\frac{\pi}{12}(t + 10.\overline{6}) = \pm 1.52 + 2\pi n$

$t + 10.\overline{6} = \pm 5.81 + 24n$



$t = -4.86$ or $-16.48 + 24n$

set rise

$\equiv 19.14$ or 7.52

$\Rightarrow$ rise @ 7:31 am and sets @ 7:08 pm

5.

a. Daily temperature follows a sinusoidal curve. In Vancouver, it reaches a minimal temperature of T_0 degrees Celsius at time t_0 and a maximal temperature of T_1 at time t_1. Determine an equation for the temperature as a function of the time t.

$$T(t) = \frac{T_1 - T_0}{2} \cos\left(\frac{\pi}{|t_1 - t_0|}(t - t_1)\right) + \frac{T_1 + T_0}{2}$$

time in hours $t=0$ is midnight

b. If $T_0 = -1$°C at 6:00 am and $T_1 = 9$°C at 7:00 pm. Then what was the temperature at 10:30 am and 10:30 pm?

$$T(t) = 5\cos\left(\frac{\pi}{13}(t-19)\right) + 4$$

$T(10.5) = 1.7°C$ @ 10:30 am ; $T(22.5) = 7.3°C$ @ 10:30 pm

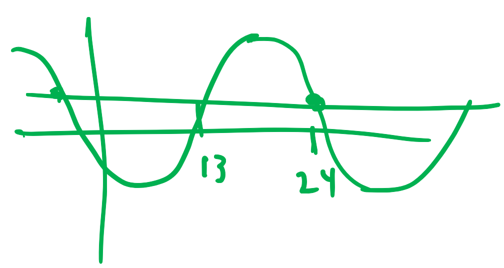
c. Determine the interval of times in the day when the temperature is above 5°C.

$$T(t) = 5 = 5\cos\left(\frac{\pi}{13}(t-19)\right) + 4$$

$$\frac{\pi}{13}(t-19) = \pm 1.37 + 2\pi n$$

$$t - 19 = \pm 5.67 + 26n$$

$$t = 24.67 \text{ or } 13.33 + 26n$$



The temp is above 5°C from 1:20 pm until a bit after midnight (12:40 am the next day)

Generalize the scenarios in the textbook page 278-280 # 17-23

17. Turn on at T_0 (min) ; turn off @ T_1 (max)
takes m min to heat up

$$T(t) = \frac{T_1 - T_0}{2} \cos\left(\frac{\pi}{m} t\right) + \frac{T_0 + T_1}{2}$$

* ask how long in a cycle is the room above/below τ degrees.

18. H_1 is max height, H_0 is min height, bounces n times in 10 seconds.

$$H(t) = \frac{H_1 - H_0}{2} \cos\left(\frac{\pi n}{5} t\right) + \frac{H_1 + H_0}{2}$$

height above the floor at time t in sec.

* ask what % of the time is the spring above or below Height h.

19.) Radius = R , T = t_0 seconds ; height get on is h (m)

$$H(t) = -R\cos\left(\frac{2\pi}{t_0} t\right) + (R+h)$$

height of the rider t sec after getting on.

* ask how long it takes to get from height h_0 to h_1.

Generalize the scenarios in the textbook page 278-280 # 17-23

20. Blade $= b$ (m) max height of Blade $= H_{max}$

makes n rotations / min

$$H(t) = b\cos(2\pi n \cdot t) + (H_{max} - b)$$

↳ t in min.

* ask for how long in a 1 min span is the blade above / below h (m).

21. Min Temp $= T_0$ on date d_0 ; max temp $= T_1$ on date d_1.

$$T(d) = \frac{T_1 - T_0}{2}\cos\left(\frac{\pi}{|d_1 - d_0|} \cdot d\right) + \frac{T_1 + T_0}{2}$$

* ask which day the temp was between τ_1 and τ_2 ▸ here $d = 1$ is Jan 1st and $d = 0$ is Dec 31st.

22. period $= T$ (years) max return $= P_1$ in year t_1, min return of P_0

$$P(t) = \frac{P_1 - P_0}{2}\cos\left(\frac{2\pi}{T}(t - t_1)\right) + \frac{P_1 + P_0}{2}$$

* ask what years they have a positive return.

Generalize the scenarios in the textbook page 278-280 # 17-23

23. make n turns in a t_0 interval

distance of a turn is r (m)

$$H(t) = r \sin\left(\frac{2\pi n}{t_0} \cdot t\right)$$

↓

horiz. distance of skier away from midline

☆ ask how a new skiers parameters would be different vs a vetern.