Week 4 Derivative Rules Review

1.

x	3	7
$h(x)$	7	22
$h'(x)$	5	10

Selected values of the increasing function h and its derivative h' are shown in the table above. If g is a differentiable function such that $h(g(x)) = x$ for all x, what is the value of $g'(7)$?

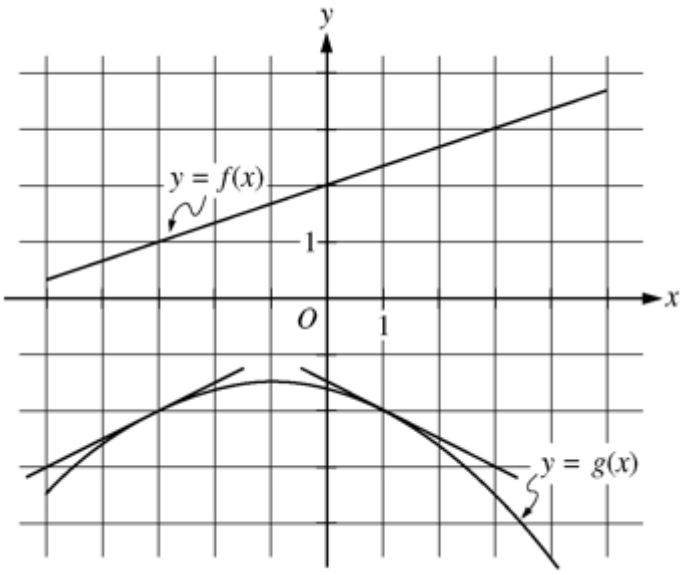
(A) $-1/10$

(B) $1/10$

(C) $1/5$ ✓

(D) $7/5$

2.



The figure above shows the graphs of the functions f and g. The graphs of the lines tangent to the graph of g at $x = -3$ and $x = 1$ are also shown. If $B(x) = g(f(x))$, what is $B'(-3)$?

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(A) $-\frac{1}{2}$

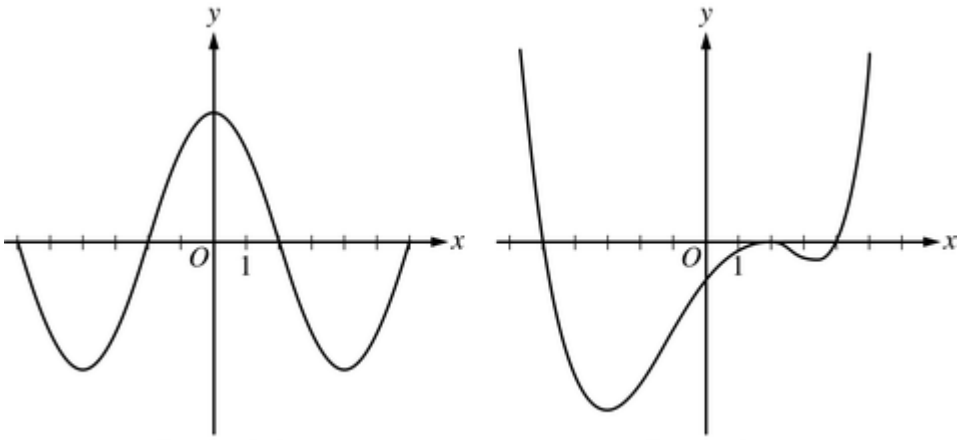
(B) $-\frac{1}{6}$ ✓

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) $\frac{1}{2}$

3.



Graph of f Graph of g

The graphs of two differentiable functions f and g are shown above. Given $p(x) = f(x)\,g(x)$ which of the following statements about $p'(-2)$ is true?

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(A) $p'(-2) < 0$ ✓

(B) $p'(-2) = 0$

(C) $p'(-2) > 0$

(D) $p'(-2)$ is undefined.

(E) There is not enough information given to conclude anything about $p'(-2)$.

4. If $y = \arctan\left(e^{2x}\right)$, then $\frac{dy}{dx} =$

(A) $\frac{2e^{2x}}{\sqrt{1-e^{4x}}}$

(B) $\frac{2e^{2x}}{1+e^{4x}}$ ✓

(C) $\frac{e^{2x}}{1+e^{4x}}$

(D) $\frac{1}{\sqrt{1-e^{4x}}}$

(E) $\frac{1}{1+e^{4x}}$

5. If g is the function given by $g(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 70x + 5$, on which of the following intervals is g decreasing?

(A) $(-\infty, -10)$ and $(7, \infty)$

(B) $(-\infty, -7)$ and $(10, \infty)$

(C) $(-\infty, 10)$

(D) $(-10, 7)$ ✓

(E) $(-7, 10)$

6. Let f be the function given by $f(x) = \cos(2x) + \ln(3x)$. What is the least value of x at which the graph of f changes concavity?

(A) 0.56

(B) 0.93 ✓

(C) 1.18

(D) 2.38

(E) 2.44

7. If $f(x) = \frac{5-x}{x^3+2}$, then $f'(x) =$

(A) $\frac{-4x^3+15x^2-2}{(x^3+2)^2}$

(B) $\frac{-2x^3+15x^2+2}{(x^3+2)^2}$

(C) $\frac{2x^3-15x^2-2}{(x^3+2)^2}$ ✓

(D) $\frac{4x^3-15x^2+2}{(x^3+2)^2}$

8. Let $f(x) = \int_0^{x^2} \sin t \, dt$. At how many points in the closed interval $[0, \sqrt{\pi}]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

(A) Zero

(B) One

(C) Two ✓

(D) Three

(E) Four

9.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	3	4	2	π

The table above gives values of the differentiable functions f and g and their derivatives at $x = 0$. If $h(x) = \frac{f(x)}{g(x)}$, what is the value of $h'(0)$?

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(A) $\frac{8-3\pi}{4}$ ✓

(B) $\frac{3\pi-8}{4}$

(C) $\frac{4}{\pi}$

(D) $\frac{2-3\pi}{2}$

(E) $\frac{8+3\pi}{4}$

10. If y=tanx−cotx, then dy/dx =

(A) sec x csc x

(B) sec x- csc x

(C) sec x + csc x

(D) $\sec^2 x$ - $\csc^2 x$

(E) $\sec^2 x$ + $\csc^2 x$ ✓

11. An equation of the line tangent to the graph of $f(x) = x(1-2x)^3$ at the point (1,–1) is

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(A) $y = -7x + 6$ ✓

(B) $y = -6x + 5$

(C) $y = -2x$

(D) $y = 2x - 3$

(E) $y = 7x - 8$

12. If $f(x) = \ln x$, then $\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$ is

(A) $\frac{1}{3}$ ✓

(B) e^3

(C) ln 3

(D) nonexistent

13. If $\ln(2x + y) = x + 1$, then $\frac{dy}{dx} =$

(A) -2

(B) $2x + y - 2$ ✓

(C) $2x + y$

(D) $4x + 2y - 2$

(E) $y - \frac{y}{x}$

14. Suppose that f is an odd function; i.e., $f(-x)=-f(x)$ for all x. Suppose that $f'(x_0)$ exists. Which of the following must necessarily be equal to $f'(-x_0)$?

(A) $f'(x_0)$ ✓

(B) $-f'(x_0)$

(C) $\frac{1}{f'(x_0)}$

(D) $-\frac{1}{f'(x_0)}$

(E) None of the above

15. If $y = x^2 e^x$, then $\frac{dy}{dx} =$

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(A) $2xe^x$

(B) $x\left(x+2e^x\right)$

(C) $xe^x\left(x+2\right)$ ✓

(D) $2x+e^x$

(E) $2x+e$

16. If $x^2+xy-3y=3$, then at the point (2, 1), $\frac{dy}{dx}=$

(A) 5 ✓

(B) 4

(C) $\frac{7}{3}$

(D) 2

17. If $f\left(x\right)=x^2+2x$, then $\frac{d}{dx}\left(f\left(\ln x\right)\right)=$

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(A) $\frac{2\ln x+2}{x}$ ✓

(B) $2x\ln x+2x$

(C) $2\ln x+2$

(D) $2\ln x+\frac{2}{x}$

(E) $\frac{2x+2}{x}$

18. If $f(x)=(x-1)^2\sin x$, then $f'(0)=$

(A) -2

(B) -1

(C) 0

(D) 1 ✓

(E) 2

19. If f is a differentiable function and $y=\sin\left(f\left(x^2\right)\right)$ what is $\frac{dy}{dx}$ when $x=3$?

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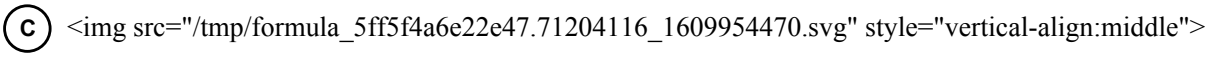
A $cos\ (f'(9))$

B $6cos\ (f(9))$

C 

D \(6f'\left(9\right)\cos\left(f\left(9\right)\right)\) ✓

20. Let *f* be the function defined by $f(x) = 2x + e^x$. If $g(x) = f^{-1}(x)$ for all *x* and the point (0,1) is on the graph of *f*, what is the value of $g'(1)$?

A $\frac{1}{2+e}$

B $\frac{1}{3}$ ✓

C $\frac{1}{2}$

D 3

E $2 + e$

21. Let *f* be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

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(A) $\frac{1}{13}$

(B) $\frac{1}{4}$ ✓

(C) $\frac{7}{4}$

(D) 4

(E) 13

22. Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\frac{1}{2}$ ✓

(B) $-\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

23. $\frac{d}{dx}\left(\tan^{-1}x + 2\sqrt{x}\right) =$

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(A) $-\frac{1}{\sin^2 x}+\frac{1}{\sqrt{x}}$

(B) $\frac{1}{\sqrt{1-x^2}}-4\sqrt[3]{x}$

(C) $\frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{x}}$

(D) $\frac{1}{1+x^2}-4\sqrt[3]{x}$

(E) $\frac{1}{1+x^2}+\frac{1}{\sqrt{x}}$ ✓

24. If $\arcsin x=\ln y$, then $\frac{dy}{dx}=$

(A) $\frac{y}{\sqrt{1-x^2}}$ ✓

(B) $\frac{xy}{\sqrt{1-x^2}}$

(C) $\frac{y}{1+x^2}$

(D) $e^{\arcsin x}$

(E) $\frac{e^{\arcsin x}}{1+x^2}$

25. The function *h* is given by $h(x)=x^5+3x-2$ and $h(1)=2$. If h^{-1} is the inverse of *h*, what is the value of $\left(h^{-1}\right)'(2)$?

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(A) $\frac{1}{83}$

(B) $\frac{1}{8}$ ✓

(C) $\frac{1}{2}$

(D) 1

(E) 8

26. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?

(A) 2 ✓

(B) $\frac{1}{2}$

(C) 0

(D) $-\frac{1}{2}$

(E) -2

27. If $\lim_{h \to 0} \frac{\arcsin(a+h) - \arcsin(a)}{h} = 2$, which of the following could be the value of a?

A $\frac{\sqrt{2}}{2}$

B $\frac{\sqrt{3}}{2}$ ✓

C $\sqrt{3}$

D $\frac{1}{2}$

E 2

28. An equation for a tangent to the graph of <img src="/tmp/formula_5ff5f4a68aa928.02401945_1609954470.svg" style="vertical-align:middle"> at the origin is

A <img src="/tmp/formula_5ff5f4a6ebfe24.90581379_1609954470.svg" style="vertical-align:middle"> ✓

B <img src="/tmp/formula_5ff5f4a7010f72.87079125_1609954471.svg" style="vertical-align:middle">

C <img src="/tmp/formula_5ff5f4a709dd79.83079161_1609954471.svg" style="vertical-align:middle">

D <img src="/tmp/formula_5ff5f4a712e071.63245032_1609954471.svg" style="vertical-align:middle">

E <img src="/tmp/formula_5ff5f4a71bf5b0.14013536_1609954471.svg" style="vertical-align:middle">

29. If $\frac{dy}{dx} = x^4 - 2x^3 + 3x - 1$, then $\frac{d^3y}{dx^3}$ evaluated at x=2 is

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(A) 11

(B) 24 ✓

(C) 26

(D) 125

30. If $x^2+y^2=25$, what is the value of $\frac{d^2y}{dx^2}$ at the point (4,3) ?

(A) $-\frac{25}{27}$ ✓

(B) $-\frac{7}{27}$

(C) $\frac{7}{27}$

(D) $\frac{3}{4}$

(E) $\frac{25}{27}$