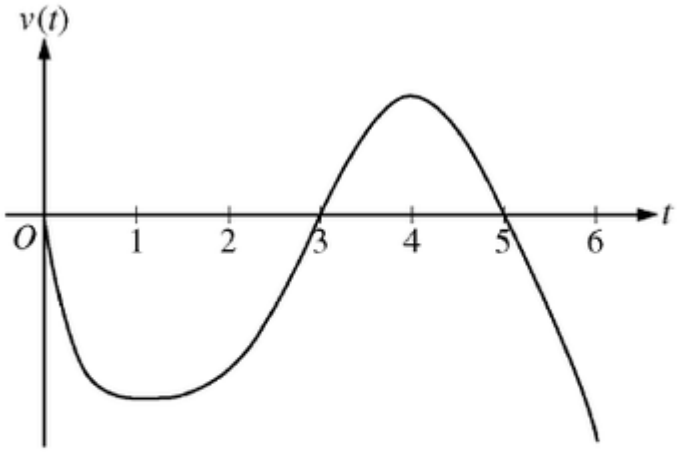
Week 5 Motion Review

Name



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals [0,3], [3,5], and [5,6] are 8, 3, and 2 respectively. At time $t = 0$, the particle is $x = -2$.

1. During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

Please respond on separate paper, following directions from your teacher.

2. On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

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Particle X moves along the positive x-axis so that its position at time $t \geq 0$ is given by $x(t) = 5t^3 - 9t^2 + 7$

3. Is particle X moving toward the left or toward the right at time $t = 1$? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

4. A second particle, Y, moves along the positive y-axis so that its position at time t is given by $y(t) = 7t + 3$. At any time $t, t \geq 0$, the origin and the positions of the particles X and Y are the vertices of a triangle in the first quadrant. Find the rate of change of the area of the triangle at time $t = 1$. Show the work that leads to your answer.

Please respond on separate paper, following directions from your teacher.

5. At what time $t \geq 0$ is particle X farthest to the left? Justify your answer.

Please respond on separate paper, following directions from your teacher.

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	–220	150

Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

6. Bob is riding his bicycle along the same path. For $0 \leq t \leq 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.

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Please respond on separate paper, following directions from your teacher.

7. Use the data in the table to estimate the value of $v'(16)$.

Please respond on separate paper, following directions from your teacher.

8. Based on the model *B* from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

Please respond on separate paper, following directions from your teacher.

9. Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\,dt$ in the context of the problem. Approximate the value of $\int_0^{40} |v(t)|\,dt$ using a right Riemann sum with the four subintervals indicated in the table.

Please respond on separate paper, following directions from your teacher.

A particle moves along the *x*-axis with position at time *t* given by $x(t) = e^{-t}\sin t$ for $0 \le t \le 2\pi$.

10. Find the time *t* at which the particle is farthest to the left. Justify your answer.

Please respond on separate paper, following directions from your teacher.

11. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

t (hours)	0	1	2	3	4
$B(t)$ (miles per hour)	1	8	1.5	-5	11

Brandon and Chloe ride their bikes for 4 hours along a flat, straight road. Brandon's velocity, in miles per hour, at time t hours is given by a differentiable function B for $0 \le t \le 4$. Values of $B(t)$ for selected times t are given in the table above. Chloe's velocity, in miles per hour, at time t hours is given by the piecewise function C defined by

$$C(t) = \begin{cases} te^{4-t^2} & \text{for } 0 \le t \le 2 \\ 12 - 3t - t^2 & \text{for } 2 < t \le 4. \end{cases}$$

(a) How many miles did Chloe travel from time $t = 0$ to time $t = 2$?

Please respond on separate paper, following directions from your teacher.

(b) At time $t = 3$, is Chloe's speed increasing or decreasing? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

(c) Is there a time t, for $0 \le t \le 4$, at which Brandon's acceleration is equal to 2.5 miles per hour per hour? Justify your answer.

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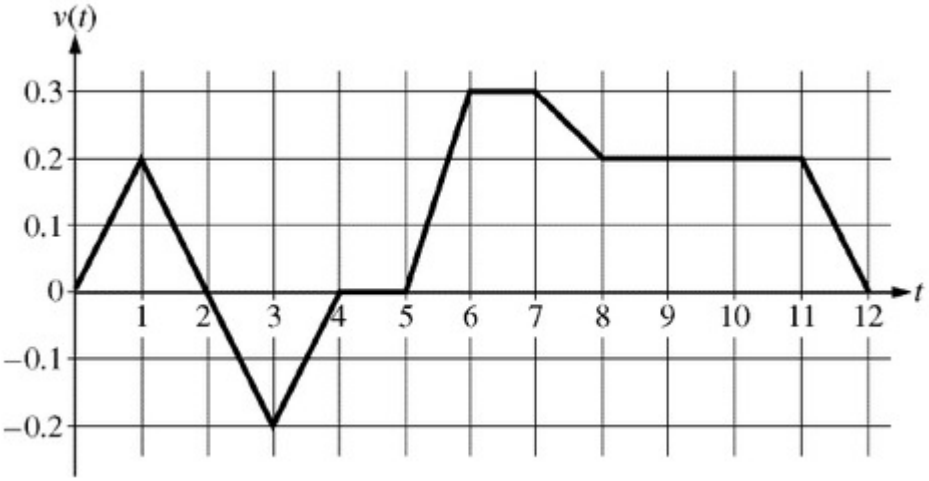
(d) Is there a time t, for $0 \leq t \leq 2$, at which Brandon's velocity is equal to Chloe's velocity? Justify your answer.

Please respond on separate paper, following directions from your teacher.



Caren rides her bicycle along a straight road from home to school, starting at home at time t=0 minutes and arriving at school at time t=12 minutes. During the time interval $0 \leq t \leq 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

12. Find the acceleration of Caren's bicycle at time t=7.5 minutes. Indicate units of measure.

Please respond on separate paper, following directions from your teacher.

13. Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

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14. Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t)=\pi/15\sin(\pi/12t)$, where w(t) is in miles per minute for $0 \le t \le$ 12 minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

Please respond on separate paper, following directions from your teacher.

15. Using correct units, explain the meaning of $\int_0^{12} |v(t)|dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|dt$.

Please respond on separate paper, following directions from your teacher.

t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
$v(t)$ (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of t over the interval $0 \le t \le 2.4$, as shown in the table above, For $0 < t \le 2.4, v(t) > 0$.

16. According to the model, $g(t) = \frac{24t+5\sin(6t)}{t+0.7}$, is Ruth's speed increasing or decreasing at time $t=1.3$? Give a reason reason for your answer.

Please respond on separate paper, following directions from your teacher.

17. Use the data in the table to approximate Ruth's acceleration at time $t=1.4$ hours. Show the computations that lead to your answer. Indicate units of measure.

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Please respond on separate paper, following directions from your teacher.

18. Using correct units, interpret the meaning of $\int_0^{2.4} v(t)\,dt$ in the context of the problem. Approximate $\int_0^{2.4} v(t)\,dt$ using a midpoint Riemann sum with three subintervals of equal length and values from the table.

Please respond on separate paper, following directions from your teacher.

19. For $0 \le t \le 4$ hours, Ruth's velocity can be modeled by the function g given by $g(t) = \frac{24t + 5\sin(6t)}{t + 0.7}$. According to the model, what was Ruth's average velocity during the time interval $0 \le t \le 4$?

Please respond on separate paper, following directions from your teacher.

20. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

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t (seconds)	0	3	8	11
$v(t)$ (meters per second)	-15	-12	-8	-3

The velocity of a particle, P, moving along the x-axis is modeled by a differentiable function v, where time t is measured in seconds and $v(t)$ is measured in meters per second. Selected values of $v(t)$ are shown in the table above.

(a) Use the data in the table to approximate $v'(10)$ using the average rate of change of $v(t)$ over the interval $8 \leq t \leq 11$. Show the computations that lead to your answer. Indicate units of measure.

Please respond on separate paper, following directions from your teacher.

(b) Interpret the meaning of $v'(10)$ in the context of the problem.

Please respond on separate paper, following directions from your teacher.

(c) Justify why there must be a time $t = k$, for $0 \leq k \leq 3$, when the velocity of the particle is -13 meters per second.

Please respond on separate paper, following directions from your teacher.

(d) Use a right Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{11} v(t)\,dt$. Show the computations that lead to your answer.

Please respond on separate paper, following directions from your teacher.

(e) Find $\int_4^{14} v'\left(\frac{t}{2}+1\right)dt$. Show the computations that lead to your answer.

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Please respond on separate paper, following directions from your teacher.

(f) Let $h(x) = \int_3^{\frac{11}{2}x} v(2t)\,dt$. Find $h'(1)$. Show the computations that lead to your answer.

Please respond on separate paper, following directions from your teacher.

(g) The position of a second particle, Q, can be modeled by a twice-differentiable function g. It is known that $g(2) = -5.5$, $g'(2) = 4$, and $g''(2) = 3$. Is the speed of particle Q increasing or decreasing at time $t = 2$? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

(h) Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = y^2 - xy$ with initial condition $f(3) = 2$. Write an equation for the line tangent to the graph of f at the point $(3, 2)$.

Please respond on separate paper, following directions from your teacher.