Week 7 Linearization

Name

1. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Consider the differential equation $\frac{dy}{dx} = 10 - 2y$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 0$. Use the tangent line to approximate $f(0.5)$.

Please respond on separate paper, following directions from your teacher.

(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(0, 2)$. Is the graph of $y = f(x)$ concave up or concave down at the point $(0, 2)$? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 2$.

Please respond on separate paper, following directions from your teacher.

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(d) For the particular solution $y = f(x)$ found in part (c), find $\lim_{x \to \infty} f(x)$.

Please respond on separate paper, following directions from your teacher.

Let f be the function defined by $f(x)=(1+\tan x)^{3/2}$ for $-\pi/4 < x < \pi/2$.

2. Using the equation found in part (a), approximate $f(0.02)$.

Please respond on separate paper, following directions from your teacher.

The following are related to this scenario:

Let f be a function with $f(2) = -8$ such that for all points (x,y) on the graph of f, the slope given by $\frac{3x^2}{y}$.

3. Write an equation of the line tangent to the graph of f at the point where $x = 2$ and use it to approximate $f(1.8)$.

Please respond on separate paper, following directions from your teacher.

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time

interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

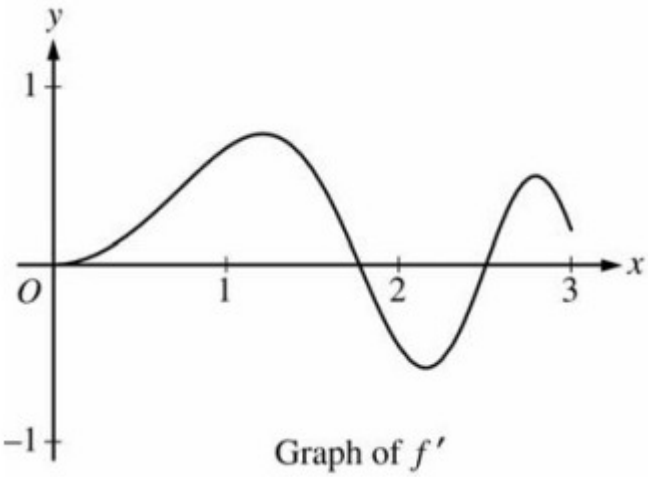
4. Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

Please respond on separate paper, following directions from your teacher.

The function g is defined for $x>0$ with $g(1)=2$, $g'(x)= \sin(x+1/x)$, and $g''(x)=(1-1/x^2)\cos(x+1/x)$.

5. Does the line tangent to the graph of g at $x=0.3$ lie above or below the graph of g for $0.3<x<1$? Why?

Please respond on separate paper, following directions from your teacher.



Graph of f'

Let f be the function defined for $x \ge 0$ with $f(0)=5$ and f', the first derivative of f, given by $f'(x)= e^{(-x/4)}\}\sin(x^2)$. The graph of $y = f'(x)$ is shown above.

6. Write an equation for the line tangent to the graph of f at $x=2$.

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Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$.

7. Write an equation for the line tangent to the curve at the point $(4, -1)$.

Please respond on separate paper, following directions from your teacher.

8. There is a number k so that the point $(4.2, k)$ is on the curve. Using the tangent line found in part (b), approximate the value of k.

Please respond on separate paper, following directions from your teacher.

9. (d) Find $\lim_{x \to -2} \frac{f(x)+7}{e^{3x+6}-1}$.

Please respond on separate paper, following directions from your teacher.

(c) For each of $\lim_{x \to 0^-} g'(x)$ and $\lim_{x \to 0^+} g'(x)$, find the value or state that it does not exist.

Please respond on separate paper, following directions from your teacher.

(b) Find the value of x in the closed interval $[-4, 3]$ at which f attains its maximum value. Justify your answer.

Please respond on separate paper, following directions from your teacher.

CALCULUS AB

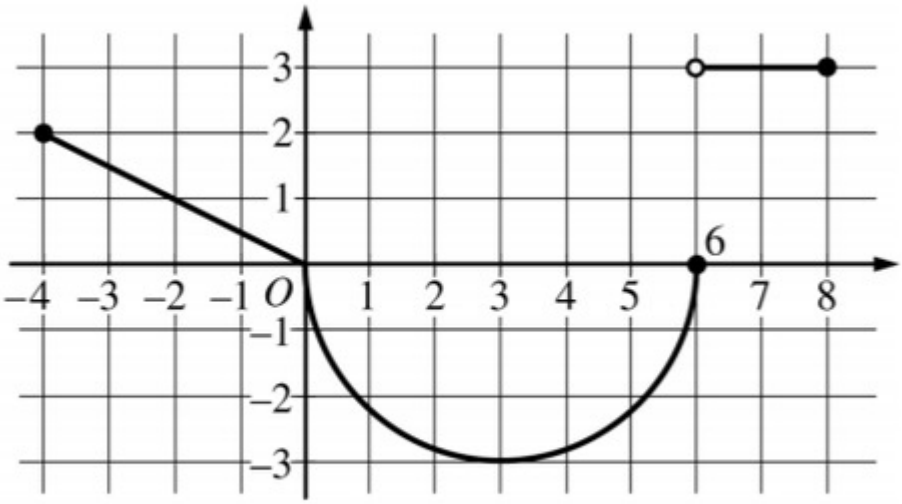
SECTION II, Part B

Time - 1 hour

Week 7 Linearization

Number of questions - 4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of g

3. The function g is defined on the closed interval $[-4, 8]$. The graph of g consists of two linear pieces and a semicircle, as shown in the figure above. Let f be the function defined by $f(x) = 3x + \int_0^x g(t)dt$.

(a) Find $f(7)$ and $f'(7)$.

Please respond on separate paper, following directions from your teacher.

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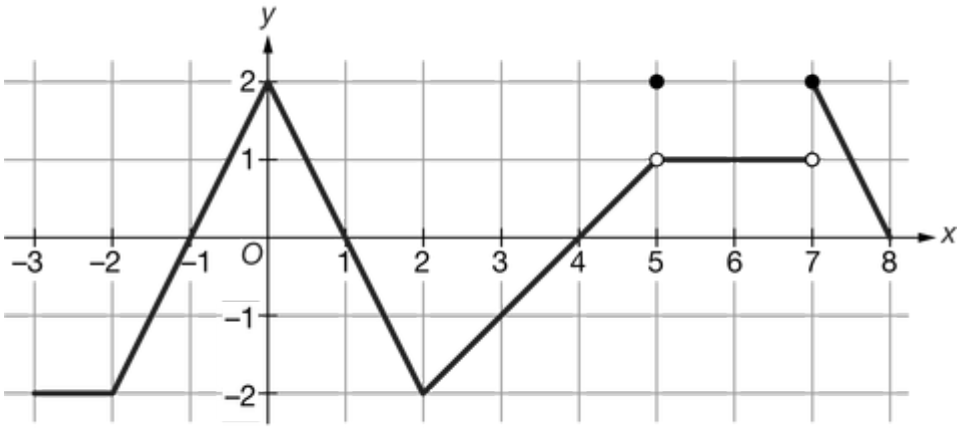

Graph of f

The graph of the function f on the closed interval $-3 \le x \le 8$ consists of six line segments and the point $(5, 2)$, as shown in the figure above. The function g is given by $g(x) = \frac{1}{10}(4x^3 + 3x^2 - 10x - 17)$. It is known that $\int_{-3}^{-1} g(x)\,dx = -4.8$ and $\int_{-3}^{4} g(x)\,dx = 11.2$.

(a) Find the value of $\int_{4}^{8} f(x)\,dx$, or explain why the integral does not exist.

Please respond on separate paper, following directions from your teacher.

(b)

(i) Find the value of $\int_{-1}^{4} g(x)\,dx$. Show the work that leads to your answer.

(ii) Find the value of $\int_{-1}^{4} (2g(x) - 4f(x))\,dx$. Show the work that leads to your answer.

Please respond on separate paper, following directions from your teacher.

(c) Let $h(x) = \begin{cases} g(x) & \text{for } x \le -1 \\ f(x) + b & \text{for } x > -1 \end{cases}$. Find the value of b for which $\int_{-3}^{4} h(x)\,dx = 14.2$.

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Please respond on separate paper, following directions from your teacher.

(d) Find $\lim_{x \to 1} \frac{f(x)}{g(x)+2}$. Show the work that leads to your answer.

Please respond on separate paper, following directions from your teacher.

The function g is continuous for all real numbers x and is defined by

$$g(x) = \frac{\cos(2x)-1}{x^2} \text{ for } x \neq 0.$$

11. Use L'Hospital's Rule to find the value of $g(0)$. Show the work that leads to your answer.

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Particle P moves along the y-axis so that its position at time t is given by $y(t) = 4t - \frac{2}{3}$ for all times t. A second particle, particle Q, moves along the x-axis so that its position at time t is given by

$x(t) = \frac{\sin(\pi t)}{2-t}$ for all times $t \neq 2$.

(a) As time t approaches 2, what is the limit of the position of particle Q ? Show the work that leads to your answer.

Please respond on separate paper, following directions from your teacher.

(b) Show that the velocity of particle Q is given by $v_Q(t) = \frac{2\pi\cos(\pi t) - \pi t\cos(\pi t) + \sin(\pi t)}{(2-t)^2}$ for all times $t \neq 2$.

Please respond on separate paper, following directions from your teacher.

(c) Find the rate of change of the distance between particle P and particle Q at time $t = \frac{1}{2}$. Show the work that leads to your answer.

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A particle moves along the y-axis so that its position at time t is given by $y(t) = t^2\tan\left(\frac{1}{t}\right)$ for $t > 1$.

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(a) Show that the velocity of the particle at time t is given by $v(t) = 2t\tan\left(\frac{1}{t}\right) - \sec^2\left(\frac{1}{t}\right)$ for $t > 1$.

Please respond on separate paper, following directions from your teacher.

(b) At time $t = \frac{4}{\pi}$, is the particle moving toward the origin or away from the origin? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

(c) The velocity of the particle at time t can be written as $v(t) = \frac{2\tan\left(\frac{1}{t}\right)}{\frac{1}{t}} - \sec^2\left(\frac{1}{t}\right)$ for $t > 1$.

Find $\lim_{t\to\infty} v(t)$. Show the work that leads to your answer.

Please respond on separate paper, following directions from your teacher.

14. Let f and g be functions that are differentiable for all real numbers, with $g(x) \neq 0$ for $x \neq 0$. If $\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = 0$ and $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x\to 0} \frac{f(x)}{g(x)}$ is

(A) 0

(B) $\frac{f'(x)}{g(x)}$

(C) $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$

(D) $\frac{f'(x)g(x) - f(x)g'(x)}{(f(x))^2}$

(E) nonexistent

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15. $\lim_{t \to 0} \frac{\sin t}{\ln(2e^t - 1)} =$

(A) -1

(B) 0

(C) $\frac{1}{2}$

(D) 1

16. Let g be a continuously differentiable function with $g(1) = 6$ and $g'(1) = 3$. What is $\lim_{x \to 1} \frac{\int_1^x g(t)\,dt}{g(x) - 6}$?

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

(E) The limit does not exist.